ECON 521, Discussion Section 10

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1. Suppose that you are attempting to buy a house and you are bargaining with the current owner over the sale price. The house is of value 1 to you and 0 to the current owner and this is common knowledge. Assume that bargaining takes place with alternating offers and that each stage of bargaining (an offer and a response) take a full day to complete. If agreement is not reached after ten days of bargaining, then the opportunity for the sale disappears (you will have no house and the current owner has to keep the house forever). Suppose that you and the current owner discount the future according to the discount factor δ per day. The real estate agent has allowed you to decide whether you will make the first offer – note that if you make the first offer, since there are ten days, your opponent will make the last offer. Assume that if an agent is indifferent between accepting an offer and not, he accepts. There exists a δ^* such that for all $\delta < \delta^*$, it is better for you (as the buyer) to go (first or second - you work it out) and for all $\delta > \delta^*$ it is better for you to go (first or second - opposite of the last one). Find δ^*.

   Hint: First, work out what offer will be made in the last round. Then use a backward induction argument. To do backward induction here, suppose the seller offers x in round t and the buyer is indifferent between accepting and not. Then, the trick is to find the y in round $t-1$ that makes the seller indifferent between accepting and holding out to offer x in round t. Then, do the same trick on the other side: suppose the buyer offers y in round t and the seller is indifferent between accepting and not. Find the x in round $t-1$ that makes the buyer indifferent between accepting and holding out to offer y in round t. Once you have these, you can iterate backwards from the last offer to calculate the SPE offers at each round.

   Solution: As for the starting place, note that whoever has the last offer can demand the full surplus and the other party will be indifferent between accepting and not.

   If the seller is offering x in round t, then the buyer wants to offer an amount y in round $t-1$ that makes the seller exactly indifferent between accepting and rejecting and waiting to counter in round t. That is

$$\begin{aligned} u_s(y) &= u_s(x) \\ \delta^{t-2} y &= \delta^{t-1} x \\ y &= \delta x \end{aligned}$$

   Now we do the opposite step. If the buyer is offering y in round t, then the seller should offer x in round $t-1$ such that the buyer is indifferent between accepting and waiting and countering. That is

$$\begin{aligned} u_b(x) &= u_b(y) \\ \delta^{t-2}(1-x) &= \delta^{t-1}(1-y) \\ (1-x) &= \delta(1-y) \\ x &= 1-\delta(1-y) \end{aligned}$$

   Then, iterating with those two equations from the starting place (the last round) gives us the following:

B first:

Day	Turn	Offer
10	S	1
9	B	δ
8	S	$1-\delta+\delta^2$
7	B	$\delta-\delta^2+\delta^3$
6	S	$1-\delta+\delta^2-\delta^3+\delta^4$
5	B	$\delta-\delta^2+\delta^3-\delta^4+\delta^5$
...	...	...
1	B	$\sum_{k=1}^{9}\delta^k(-1)^{k-1}$

S first:

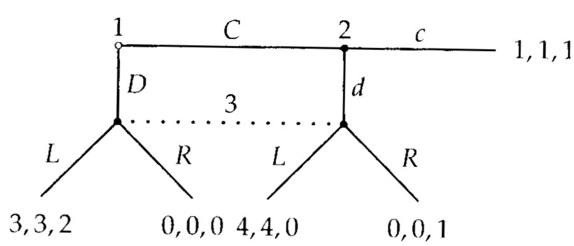
Day	Turn	Offer
10	B	0
9	S	$1-\delta$
8	B	$\delta-\delta^2$
7	S	$1-\delta+\delta^2-\delta^3$
6	B	$\delta-\delta^2+\delta^3-\delta^4$
5	S	$1-\delta+\delta^2-\delta^3+\delta^4-\delta^5$
...	...	...
1	S	$\sum_{k=0}^{9}\delta^k(-1)^k$

These are the SPE offers in each round (by backwards induction). The outcome is that they accept in the first round. The buyers payoff is 1 minus the offer. So the relevant comparison is:

$$1-\sum_{k=1}^{9}\delta^k(-1)^{k-1} \gtrless 1-\sum_{k=0}^{9}\delta^k(-1)^k$$

You can solve this and find that the LHS is larger if $\delta < 0.79593$. That is, if $\delta < 0.79593$, it's better to go first. Otherwise it's better to go second.

2. (Selten's Horse) Find the NE, the SPE, and the sequential equilibria of Selten's Horse.



Solution: First' are there any proper subgames? No! So the set of NE equals the set of SPE. Let's find those first by looking at the normal forms. Therefore the two

L	c	d
C	$1,1,\underline{1}$	$\underline{4},\underline{4},0$
D	$\underline{3},\underline{3},\underline{2}$	$3,\underline{3},\underline{2}$

R	c	d
C	$\underline{1},\underline{1},\underline{1}$	$\underline{0},0,\underline{1}$
D	$0,\underline{0},0$	$\underline{0},\underline{0},0$

NE and SPE are (D,c,L) and (C,c,R). Are these sequential equilibria? To work this out, we need to see what each would imply about P3's beliefs, $\mu(D)$ and $\mu(C,d)$ where the two sum to one.

In the equilibrium (D, c, L), whenever P3 decides, it is because D has been played. Therefore, in equilibrium, it must be that $\mu(D) = 1$ and $\mu(C, d) = 0$. Player 2, if he gets a chance to play, will then never play c, since d has a payoff of 4, while c would give him 1. Remember that in sequential equilibria, a player must behave optimally given what others are doing at each decision node, even if that decision node isn't reached. If he were to play d, then player of 1 would prefer C, but (C, d, L) is not an equilibrium, because then we would have $\mu(C, d) = 1$ and P3 would prefer R.

In the equilibrium (C, c, R), since P3's information set is not reached in equilibrium, there is no restriction regarding the beliefs he may have if his information set is reached. Therefore, we can define his beliefs arbitrarily. Of course, for him to want to play R, and to therefore support this equilibrium, it must be the case that $\mu(D) \leq 1/3$. Remember that these beliefs are an equilibrium object in sequential equilibria, so you must state them to get full credit.