ECON 521, Discussion Section 12

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1. Consider the following Bayesian game:

1 \ 2	C	D
A	x, 9	3, 6
B	6, 0	6, 9

$$x = \begin{cases} 12 & \text{with probability } 2/3 \\ 0 & \text{with probability } 1/3 \end{cases}$$

   (a) Draw the extensive form of this game, with nature making the first choice in choosing x with probabilities described above and only P1 observing nature's choice. P1 and P2 then choose between their actions simultaneously.

   (b) Draw the *ex ante* strategic form of the game.

   (c) Solve for the Bayesian Nash Equilibrium using this form.

   (d) Draw the *interim* strategic form of the game.

   (e) Solve for the Bayesian Nash Equilibrium, and confirm your answer matches that from above.

2. Consider a Cournot duopoly game with incomplete information. Demand is given by $p = 1 - Q$ where Q is total quantity. Firm 1 selects q_1, which it produces at no cost. Firm 2's cost of production (selected by nature) is private information. With probability 1/2, firm 2 produces at zero cost. With probability 1/2, firm 2 produces with marginal cost of 1/4. Call the former type of firm 2 "L" and the latter type "H" (for low and high cost, respectively). Firm 2 knows its type, whereas firm 1 knows only the probability of L and H (1/2 each). Let q_2^H and q_2^L denote the quantities selected by the two types of firm 2.

   (a) If the goal is to find the BNE in this game, why is it impossible to write the *ex ante* and *interim* strategic forms as we did in the last problem?

   (b) Your answer to (a) notwithstanding, you can use the *interim* logic to find the BNE here. To do so, treat each type as a separate player. Find the best response for each and solve for the BNE.

3. In the lecture, we have seen that in a second-price auction with two bidders whose values are drawn randomly from a uniform distribution on $[0, 1]$, there exists a symmetric BNE in which agent i bids v_i. We have also seen that in a first-price auction in the same environment, there exists a symmetric BNE in which agent i bids $v_i/2$. Further, we found that expected revenue in either case was 1/3. Now consider an all-pay auction in the same environment – that is, the highest bidder is awarded the good but both agents pay their bids.

   (a) What is the expected revenue from this auction? (Hint: No math required) Why? Given this, should a seller consider using an all-pay auction to sell the good?

   (b) Show that there exists a symmetric BNE in which each agent bids $b_i = \frac{1}{2}v_i^2$ and calculate the expected revenue to confirm that the Revenue Equivalence Theorem holds here.