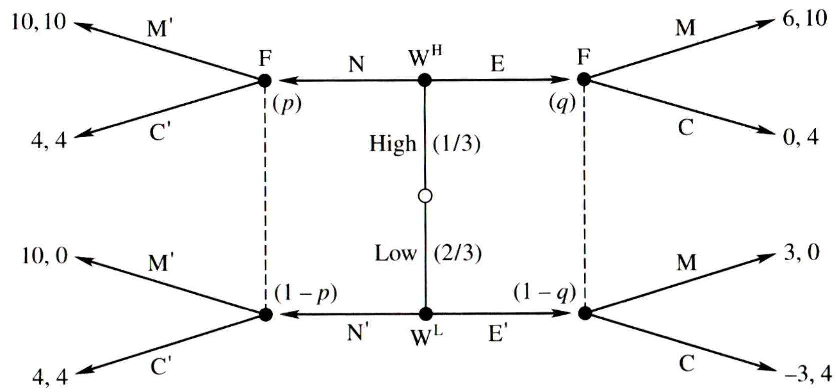
ECON 521, Discussion Section 13

TA: Shane Auerbach (sauerbach@wisc.edu) ; Date: 12/12/14

1. Consider the following job-market signaling game similar to Spence 1973 (and as presented in Watson page 350):

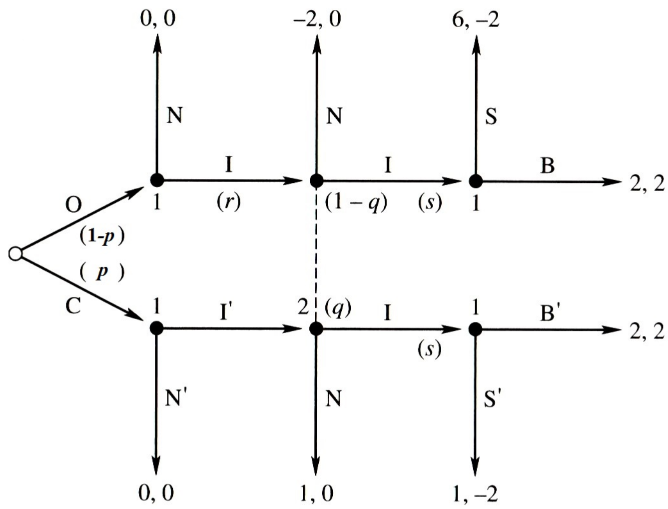


The incomplete information game above is played between a worker (W) and a firm (F). The worker has private information about her level of ability. With probability 1/3 she is a high type (H) and with probability 2/3 she is a low type (L) – these probabilities are common knowledge. After observing her own type, the worker decides whether to obtain a costly education (E) or not (N); think of E as getting a degree. The firm observes the worker's education (which is described in her resume), but the firm does not observe the worker's quality type. The firm then decides whether to emply the worked in an important managerial positions (M) in a much less important clerical job (C). In equilibrium, a firm may deduce the worker's quality level on the basis of the worker's education (or not). You can see from the payoffs that high and low type value the two jobs equally (10 and 4 for M and C respectively), however the two types have different educations costs (4 for H and 7 for C). The firm gets a payoff of 10 from putting an H worker into an M job, a payoff of 4 from putting any worker into a C job, and a payoff of zero from putting an L worker into a M job. Note that the firm's payoffs are independent of the education level of the workers. To find Perfect Bayesian Equilibria (PBE), we can look for pooling equilibria (in which both types of worker make the same decision) and separating equilibria (in which the two types of workers make different decisions).

   (a) Find the pooling equilibrium.

   (b) Find the separating equilibrium in pure strategies.

2. Consider the following investment game with incomplete information:



The story of this game as follows. P1 owns a remote-controlled drone. It's pretty sweet, but it's broken in two ways. First, the rotor is broken – fortunately P1 knows how to fix the rotor. Second, the remote-control is not working – P1 does not know how to fix it, but P2 does. If P1 invests time in fixing the rotor (I and I'), and P2 invests time in fixing the controller (I), then the drone becomes operable. The repairs must happen in that order – you don't want to test the controller (and thus the drone) when it still has a broken rotor (duh!). If the drone is operable, P1 may choose to be *benevolent* (B) and share it with P2, or she may choose to be *selfish* (S) and keep it to herself. The added complication is that P2 does not know whether P1 is *ordinary* (O) in that she would prefer not to share or *cooperative* (C) in that she would prefer to share (and actually enjoys working on the drone). In particular, there is a probability p that P1 is cooperative, and a probability $1 - p$ she is ordinary. The payoffs are listed above.

(a) Suppose it is known that P1 is the ordinary type, i.e. $p = 0$ above. Find the subpage perfect equilibrium (SPE) of the resulting complete information game.

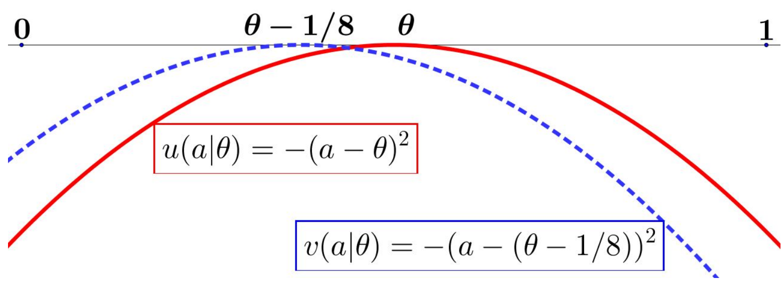
(b) Suppose it is known that P1 is the cooperative type, i.e. $p = 1$ above. Find the SPE of the resulting complete information game.

(c) Now suppose $p = 1/4$. Argue that there exists no perfect Bayesian equilibrium (PBE) in pure strategies.

(d) Still supposing $p = 1/4$. Find a PBE in mixed strategies.

3. In the previous Spence signaling game, the signal (education there) could be informative precisely because it was more costly for one type of agent than for the other. But what about when the signal is costless? We call these models of *cheap talk*, pioneered by the classic paper of Crawford and Sobel (1982). The following question is a simplification of that model:

   Define $\theta \in [0,1]$ to represent the degree to which the Hudson River is polluted. In Albany, New York, the governor asks a bureaucrat to work out how polluted the Hudson is – that is, she is asked to determine θ. The bureaucrat will then report a to the governor, and she wants to be as right as possible, so her utility is given by $u(a|\theta) = -(a-\theta)^2$. The problem is that the bureaucrat does not know θ or have any means to measure it directly. In particular, her prior belief is that all levels of $\theta \in [0,1]$ are equally likely. Fortunately, she has an expert acquaintance who knows how to measure θ. The expert, however, also consults with some big factories that pollute the river, and therefore has a slight bias in that if the true level of pollution is θ, he would prefer the governor thought it was $\theta - 1/8$. The expert's utility can therefore be represented by $v(a|\theta) = -(a-(\theta-1/8))^2$. He shares the bureaucrat's prior belief. All of the above is common knowledge to both the bureaucrat and the expert, and is represented below (the $\theta = 1/2$ case is shown):



   The expert may measure θ and then communicate (with message m) his findings in whatever way he wants to the bureaucrat.

   (a) Argue (simply) that there cannot be a PBE in which the expert reports the true θ.

   (b) Argue (simply) that there cannot be a PBE in which the expert reports $\theta - 1/8$.

   (c) There is a PBE in which the expert truthfully reports that θ is either above ($m = H$) or below ($m = L$) a certain threshold θ^* and the bureaucrat believes the report. Find θ^*.