ECON 521, Discussion Section 1 SOLUTIONS

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Questions

Q1: Are the following two games (G and G') static or dynamic? Are they displayed in the normal (strategic) form or the extended form? Which archetypical game do G and G' most resemble (pick from: Prisoner's Dilemma, Battle of the Sexes, Stag Hunt, Good Restaurant Bad Restaurant, Matching Pennies)? Are G and G' *ordinally equivalent*? Are G and G' *cardinally equivalent*?

G :

	L	R
T	19,15	3,3
B	0,0	15,19

G′ :

	L	R
T	-2,-3	-4,-4
B	-5,-5	-3,-2

Solution: The two games are static. Dynamic games can be represented in this form, but then strategies of at least one of the players would have to be contingent on another to capute the timing. Since there is no obvious contingency in the strategies here, the game is static. They are displayed in the normal (strategic) form. They resemble Battle of the Sexes, in that they like to coordinate (at TL or BR), but one of them prefers TL where the other prefers BR. They are ordinally equivalent - check that each player has the same ranking over outcomes over the two games. They are not cardinally equivalent. However, G' is actually cardinally equivalent to how BoS is often written:

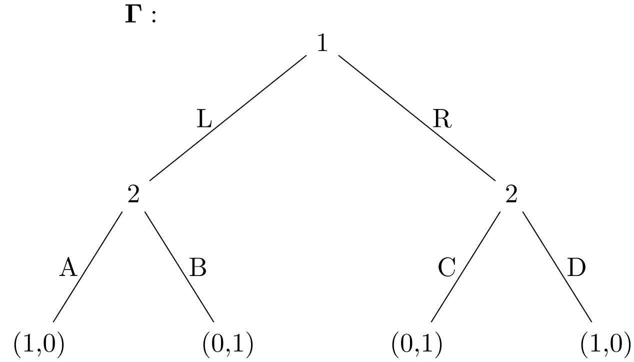
BoS :

	L	R
T	3,2	1,1
B	0,0	2,3

G' is just BoS with five subtracted from each payoff.

Q2: Is the following game, Γ, static or dynamic? Is it displayed in normal (strategic) form or extensive form? Can it be displayed in the other form? If so, do it. If not, explain why it cannot. Why is A not a strategy for player 2?

Γ :



Solution: Γ is dynamic, as 2 makes her decision after observing 1's decision. It is displayed here in extennsive form (i.e. a game tree). Of course it can be displayed in normal form, as long as we are careful about what each agent's strategies are. Agent 1 can only choose between L and R, so these are her possible strategies. For Agent 2, she can condition her choice on Agent 1's choice. Therefore, a strategy for Agent 2 describes what she would do at each of her two decision nodes. Her possible strategies are then $\{AC, AD, BC, BD\}$, where, for each strategy, the first letter indicates what she will play if Agent 1 plays L and the second letter indicates what she will play if Agent 2 plays R.

Just 'A' is not a strategy because it does not tell us what she would do if Agent 1 plays R. A is a possible action, but not a strategy. The normal form of Γ would be

	AC	AD	BC	BD
L	1,0	1,0	0,1	0,1
R	0,1	1,0	0,1	1,0

If you have a particularly good intuition for game theory, you might have also noticed that this game is really like matching pennies except one person gets to make their decision after the other has already committed. This makes it a pretty trivial game. If my goal is to match the side of your penny, and I can make my decision after you choose heads or tails, I will obviously just choose the same thing as you!

Q3: Write up the normal (strategic) form (i.e. a bi-matrix of payoffs) for *Rock, Paper, Scissors* given that a player gets one util if she wins, negative one if she loses, and zero otherwise. Is this game typically played with complete information? Is it static or dynamic?

Solution:

	r	p	s
R	0,0	0,1	1,0
P	1,0	0,0	0,1
S	0,1	1,0	0,0

Rock, Paper, Scissors is obviously static - the whole point is that both players must decide simultaneously. As for the information, it might seem that this game has incomplete or assymetric information, but it is usually played with complete information. You both know the rules. You both know you both the rules. You both know you both know you both know the rules, and etc. So we have common knowledge there. Also, while you don't know what action they will choose, you do know their preferences over outcomes. They prefer winning to drawing and drawing to losing. Therefore, we have complete information. The lesson is that you having no idea what your opponent may play might be a consequence of the strategy of the game, not a consequence of some underlying informational incompleteness or assymetry.

Q4: For each table, determine whether the two utility functions are *ordinally equivalent*, *cardinally equivalent*, both, or neither. If they are *cardinally equivalent*, write v as a transformation of u. Let O denote the outcome.

Table 1

O	$u(O)$	$v(O)$
A	1	6
B	7	18
C	2	8

Table 2

O	$u(O)$	$v(O)$
A	9	-2
B	-12	-9
C	3	-4

Table 3

O	$u(O)$	$v(O)$
A	1	4
B	2	5
C	3	7

Solution: For Table 1, the two utility functions are cardinally equivalent, since one is an affine transformation, $v = Au + B$, of the other. Here, $v = 2u + 4$. If it is cardinally equivalent, it must also be ordinally equivalent (this is always true).

For Table 2, the two utility functions are again both cardinally equivalent and ordinally equivalent. Try $v = \frac{1}{3}u - 5$. This might seem surprising as the transformation from u to v seems to make outcomes A and C worse, but seems to make B better.

For Table 3, the two utility functions are ordinally equivalent but not cardinally equivalent. To see that they are not cardinally equivalent, note that the difference between A and B in both u and v is one. This implies that if there were an affine transformation,

$v = Au + B$, A must equal 1. Given that, B must equal 3. However, $v = u + 3$ does not work for outcome C. Therefore, the two utility functions are not cardinally equivalent.

Q5: At the doctor's office, you test positive for a disease called *Gametheoryitis*, a crippling condition under which you apply game theoretic reasoning to every situation in life, even when such reasoning is unnecessary. Exactly one percent of people suffer from this affliction. Your doctor tells you that the test is sometimes incorrect. In particular, when given to people who do not suffer from this affliction, the test still yields a positive result ten percent of the time. In the other direction, five percent of those with the disease receive negative test results. Given that you tested positive, what is the probability you have the affliction? (hint: Bayes' rule)

Solution: Let A represent having the affliction and T represent testing positive. Let $\sim$ represent the word 'not'. Then

$$P(A|T) = \frac{P(A \cap T)}{P(T)} = \frac{P(T|A)P(A)}{P(T|A)P(A) + P(T| \sim A)P(\sim A)} = \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.1 \times 0.99} \approx 0.1$$

Therefore, even though you tested positive, there's still only about a 10% chance you have the affliction.