ECON 521, Discussion Section 2 SOLUTIONS

TA: Shane Auerbach (sauerbach@wisc.edu) ; Date: 9/12/14

Q1 (Silvio): Silvio Berlusconi has just received some terrible news - he turns to you as his advisor in all things game theoretic. On a recent trip to Toronto, Berlusconi had a late night out with Toronto mayor Rob Ford and not-Toronto-mayor Justin Bieber. That's not the terrible news though - he had a great time! The terrible news is that one of Toronto's newspapers, *The Globe and Mail*, has photos of the three doing cocaine behind a Tim Horton's at a truckstop at 4am. *The Globe and Mail* will either release those photos or choose not to. Silvio must decide how to handle the situation simultaneously to the newspaper's decision on whether to print or not. The newspaper has no soul, unlike poor Silvio, and therefore gets a utility of zero no matter what happens.

You and Silvio come up with four options:

W: He can go to the other paper, *The Star*, and give them an exclusive on his night-out. If he does this, it doesn't matter whether *The Globe and Mail* prints the photos or not, because everybody will read the exclusive instead. This option would be embarrassing for him, yielding him a utility of -2.

X: He can deny rumors of the night-out. If *The Globe and Mail* prints the photos, then he is revealed to be a liar and he gets a utility of -3. If the paper doesn't print them, however, he not only gets no negative utility, but he is so thrilled to get away with his lie that he gets utility of 1.

Y: He can attempt to distract the public by having his Italian media outlets publish sensational stories accusing footballer Mario Balotelli of being a unicorn sent from an alien unicorn planet. This would make his media outlets seem a little silly, but would lessen the impact of the photos, if published. His utility if the photos are published would be -2. If the photos are not published, his utility would be zero.

Z: He can buy *The Globe and Mail* and destroy the photos. Newspapers are worth nothing nowadays, and he has plenty of money. If he buys the paper, it doesn't matter what decision the paper makes - he'll destroy the photos before they could go to print - he gets a utility of -1 whatever the paper decides.

Silvio asks you to provide the following:

a) A formal description of the game, $G = \langle I, (A_i, u_i)_{i \in I} \rangle$.

b) A representation of the game in normal/strategic form.

c) A graph of the expected payoffs as a function of conjectures (similar to that on page 35 of the lecture notes, which is 41 of the pdf)

d) A state contingent representation of the expected payoffs (similar to that on page 37 of the lecture notes, which is 43 of the pdf)

e) A recommendation on actions above that Silvio should definitely avoid.

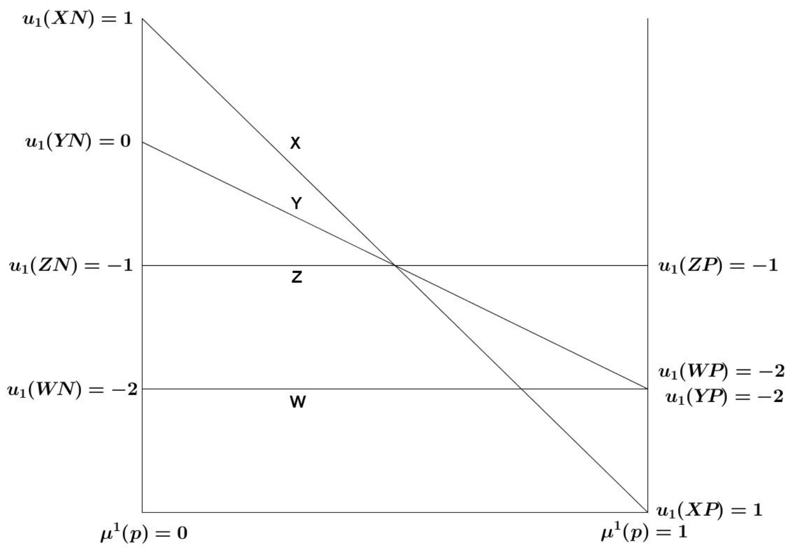
f) A recommendation for Silvio given that he thinks the probability that *The Globe and Mail* publishes the photos is 40%.

Solution: The set of players is $I = \{\text{Silvio}, \textit{The Globe and Mail}\}$. The set of actions for Silvio is $A_1 = \{W, X, Y, Z\}$ as defined above. The set of actions for *The Globe and Mail* is $A_2 = \{N, P\}$, where N represents not printing the photos and P represents printing them. The utility functions are $u_1(WN) = u_1(WP) = -2, u_1(XN) = 1, u_1(XP) = -3, u_1(YN) = 0, u_1(YP) = -2, u_1(ZN) = u_1(ZP) = -1$. For the newspaper, $u_2(\cdot) = 0$

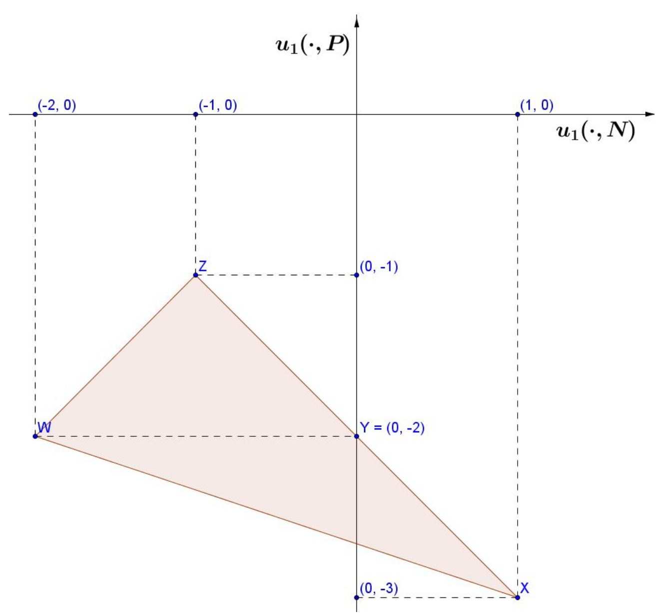
for all possible outcomes. The normal/strategic form of the game is therefore as follows.

	N	P
W	-2,0	-2,0
X	1,0	-3,0
Y	0,0	-2,0
Z	-1,0	-1,0

The expected payoffs as a function of conjectures should look like the following.



The state contingent representation of expected payoffs should like the following.



As for what Silvio should avoid, our first graph suggests that only W is not justifiable, in that it's not a best response for any conjecture. It might seem like Y is a bad choice, given that it's not the unique best response for any conjecture, but it is a best response (along with X and Z) when $\mu^1(p) = 1/2$.

Finally, given $\mu^1(p) = 0.4$, we can see from the first graph that X is the best response.

Q2 (Conjectures): Given the following game, for each vector ($\mu_1^1, \mu_2^1, \mu_3^1,$ and μ_4^1), say whether or not that vector is a possible conjecture for player 1, explaining why or why not. If it is a possible conjecture, do you believe that conjecture is consistent with player 1 believing that player 2 is rational? Finally, why do we never say that a conjecture is irrational?

	L	C	R
T	2,1	1,2	2,0
B	3,0	3,1	1,4

$$\mu_1^1 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right), \quad \mu_2^1 = \left(\frac{1}{2}, \frac{1}{2}, 0\right), \quad \mu_3^1 = (0, 0, 1), \quad \mu_4^1 = \left(0, \frac{1}{4}, \frac{1}{4}\right)$$

Solution: For any of those vectors to be conjectures, it need only be the case that each probability in the list is non-negative and that the sum of the elements equals one. Therefore, $\mu_1^1, \mu_2^1,$ and μ_3^1 are conjectures, but μ_4^1 is not. As for which are consistent with player 1 believing that player 2 is rational, note that playing L is not justifiable for player 2 because it is not the best response no matter what action (mixed or pure) player 1 chooses. It is, in fact, strictly dominated by C. Therefore, any conjecture for player 1 that puts positive probability on player 2 playing L is inconsistent with player 1 believing that player 2 is rational, i.e. μ_1^1 and μ_2^1 are inconsistent with this belief.

As for why we never call a conjecture irrational, just because μ_1^1 and μ_2^1 are inconsistent with the belief that player 2 is rational, the fact that you are rational doesn't require you to believe that others are rational. A rational agent can, perfectly reasonably, believe his opponent to be irrational. That's why μ_1^1 and μ_2^1 are still rational conjectures. As for μ_4^1, it is not an irrational conjecture - it simply isn't a conjecture!

Q3 (Mixed Actions): Given the following game (in which only player 1's payoffs are shown), come up with a mixed action for player 1, α_1, such that his expected utility is exactly (no more, no less) equal to one no matter what player 2 plays. Come up with a different mixed action for player 1 such that his expected utility is 1 if player 2 plays R and 1/2 if player 2 plays L. Is either of the mixed actions you just came up with justifiable for player 1? Why or why not?

	L	R
A	0	3
B	3	0
C	0	0

Solution:

$\alpha_1 = (\alpha_1(A), \alpha_1(B), \alpha_1(C)) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ gives an expected utility of one to player 1 regardless of what player 2 selects. To come up with that, it's sometimes easier to think of it as a compound lottery. For instance, think of it as mixing 50/50 between A and B two thirds of the time (getting expected utility of 1.5 no matter what P2 picks), and then selecting C the other third - then multiplying out gives us the answer above.

$\alpha_1' = (\alpha_1'(A), \alpha_1'(B), \alpha_1'(C)) = (\frac{1}{3}, \frac{1}{6}, \frac{1}{2})$ gives player 1 expected utility of 1 if player 2 plays R and 1/2 if player 2 plays L. Again, you can think of it as a player 1 playing a two thirds / one third mix on a and b half the time, and playing c the other half of the time.

Neither of these actions is justifiable, because neither is the best response to any conjecture player 1 might have about player 2's play. Specifically, no matter what conjecture P1 has on P2's play, it is irrational for P1 to play C with positive probability, i.e. to include C in a mix.

Q4 (Paper, Scissors, Rock, Conjectures): Consider the popular game *Paper, Scissors, Rock*. You get one util for winning, zero for tying and negative one for losing. For all possible conjectures you might have about how your opponent might play, find the best responses. Since there are infinitely many conjectures, you should derive conditions on those conjectures, and describe the best responses given those conditions.

Solution: Let P represent your opponent's action to play paper. Similarly for S and R. Then $A_1 = P, Q, R$

- If $\arg\max_{x \in A_1} \mu^1(x) = \{P\}$, then $r_1(\mu^1) = \{S\}$.
- If $\arg\max_{x \in A_1} \mu^1(x) = \{S\}$, then $r_1(\mu^1) = \{R\}$.
- If $\arg\max_{x \in A_1} \mu^1(x) = \{R\}$, then $r_1(\mu^1) = \{P\}$.
- If $\arg\max_{x \in A_1} \mu^1(x) = \{P, S\}$, then $r_1(\mu^1) = \{S\}$.
- If $\arg\max_{x \in A_1} \mu^1(x) = \{P, R\}$, then $r_1(\mu^1) = \{P\}$.
- If $\arg\max_{x \in A_1} \mu^1(x) = \{S, R\}$, then $r_1(\mu^1) = \{R\}$.
- If $\arg\max_{x \in A_1} \mu^1(x) = \{P, S, R\}$, then $r_1(\mu^1) = \{P, S, R$ or any mix of the three$\}$.

This may not be as intuitive as you would have thought, but I think it's correct. Challenge me if you disagree, or ask if you have questions.

Q5 (Independent Mixed Actions): For each of the following games, in which in each cell is listed the probability of the particular outcome, rather than payoffs, come up with an α_1 and an α_2 (which use independent randomization devices) such that the outcomes are realized with the expected probabilities, or argue that the players would need to correlate to achieve the listed probabilities.

	L	R
T	1/3	1/6
B	1/3	1/6

	L	R
T	1/3	1/6
B	1/6	1/3

	L	R
T	1/3	1/3
B	1/6	1/6

Solution: The first distribution can be achieved by $\alpha_1 = (1/2, 1/2)$ and $\alpha_2 = (2/3, 1/3)$. The second distribution would require correlation (try to set up a system of equations to find the α's and show a contradiction). The third distribution can be achieved by $\alpha_1 = (2/3, 1/3)$ and $\alpha_2 = (1/2, 1/2)$