

**ECON 521, Discussion Section 3**

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**Q1 (Properties of the Rationalization Operator,  $\rho$ )**

Consider the extended Battle of The Sexes Game below as an example.

**Extended BoS :**

		b	s	c
B	4,3	0,2	0,0	
S	0,1	3,4	0,0	
C	1,1	1,2	5,0	

State whether each of the following statements is true or false and explain using the game above.

$a : \forall C \subseteq A, \rho(C) \subseteq (C)$

$b : \forall C \subseteq A, |\rho(C)| \leq |C|$  (you may need to use a different game to show this one)

$c : \forall i \in \mathbb{N}, \rho^i(A) \subseteq \rho^{i-1}(A)$

$d : a_i$  is rationalizable if and only if  $\forall C \subseteq A$  such that  $a_i \in C, \rho(C) = C$

**Q2 (Osborne & Rubinstein Exercise 64.1):** Each of two players announces a nonnegative integer equal to at most 100. If  $a_1 + a_2 \leq 100$ , then each player  $i$  receives a payoff of  $a_i$ . If  $a_1 + a_2 > 100$ , and  $a_i < a_j$ , then player  $i$  receives payoff of  $a_i$  and player  $j$  receives  $100 - a_i$ ; if  $a_1 + a_2 > 100$  and  $a_i = a_j$  then each player receives 50. Show that the game is dominance solvable and find the set of surviving outcomes. Here, you may eliminate weakly dominated strategies in solving the game, though this is dicey in general.

**Q3 (Osborne Exercise 372.9):** Give an example of an action that is a best response to a belief but is not a best response to any belief that assigns probability 1 to a single action. One way to do this is to design your own static game such that this is true.

**Q4 (Watson Exercise 6.1):** Find the set of rationalizable actions for each player in **G**.

**G :**

		W	X	Y	Z
U	3,6	4,10	5,0	0,8	
M	2,6	3,3	4,10	1,1	
D	1,5	2,9	3,0	4,6	

**Q5 (Hotelling's Model of Electoral Competition):** Suppose there are two candidates in an election and the set of possible positions they can take is  $\{0, 1, 2, \dots, l\}$ , where  $l$  is even. Each voter has his own position, and will vote for whichever candidate is closest to his position. Suppose there exists a position  $m$  such that exactly half of the voters' positions are at least  $m$  and exactly half of the voters' positions are at most  $m$ . Each candidate wants to win the election by getting the most votes – they only care about winning, not how many votes they get. If the two candidates get the same number of votes, they both lose. What actions survive iterated deletion of weakly dominated strategies?

**Q6 (Watson Exercise 8.7):** Consider a game in which, simultaneously, player 1 selects a number  $x \in [2, 8]$  and player 2 selects a number  $y \in [2, 8]$ . The payoffs are given by

$$u_1(x, y) = 2xy - x^2$$

$$u_2(x, y) = 4xy - y^2$$

Calculate the justifiable and rationalizable actions for each player in this game.