ECON 521, Discussion Section 4

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1. Find all mixed strategy equilibria (including pure-strategy NE) of the following games:

	L	M	R
U	$8,3$	$3,5$	$6,3$
C	$3,3$	$5,5$	$4,8$
D	$5,2$	$3,7$	$4,9$

G

	L	M	R
U	$8,1$	$0,2$	$4,3$
C	$3,1$	$4,4$	$0,0$
D	$5,0$	$3,3$	$1,4$

G′

2. Consider the following social problem. A pedestrian is hit by a car and lies injured on the road. There are n people in the vicinity of the accident. The injured pedestrian requires immediate medical attention, which will be forthcoming if at least one of the n people calls for help. Simultaneously and independently each of the n bystanders decides whether or not to call for help (by dialing 911). Each bystander obtains v units of utility if anyone calls for help. Those who call for help pay a personal cost of c. That is, if person i calls for help, then he obtains the payoff $v-c$; if person i does not call but at least one other person calls, then person i gets v; finally, if none of the n people calls for help, then person i obtains zero. Assume $v > c$.

   (a) Describe the NE in pure strategies.

   (b) Find the symmetric mixed equilibria (i.e. all players do the same). In your analysis, let p be the probability that a person doesn't call for help.

   (c) Compute the probability that at least one person calls for help in the mixed equilibrium. Comment on how this depends on n and whether the result is intuitive?

3. Consider the following 3-player game where P1 chooses the row, P2 chooses the column, and P3 chooses the bimatrix (A or B). P1's payoff is listed first, P2's is listed second, and P3's is listed third. Find the pure-strategy NE.

	L	R
U	$2,0,2$	$1,1,1$
D	$1,1,1$	$0,2,2$

A

	L	R
U	$0,2,2$	$1,1,1$
D	$1,1,1$	$2,0,2$

B

4. The Allies have one bomber plane which they can use to strike one of three possible targets: a dam (D), an airstrip (A) and a tank (T). The values of those targets are $v_D = 4, v_A = 3, v_T = 2$. The Axis has one anti-aircraft gun, and can choose to defend only one of the three targets. The bomber destroys the target if it is undefended, and does no damage if it is defended. The Allies get utility equal to the value of the object they destroy, if they destroy an object. The Axis gets utility equal to negative the value of an object destroyed, if an object is destroyed. As an example, listing the Allies choice first, $u_{Allies}(D,T) = 4, u_{Axis}(D,T) = -4$. $u_{Axis}(D,D) = u_{Allies}(D,D) = 0$. The Allies and Axis make their decisions simultaneously. Find a mixed equilibrium in which both the Axis and Allies mix across all three of their actions.

(From lecture notes pages 80-81)

A general Algorithm (semi-formal). In two-player games with finite strategy sets, one can find all (pure and mixed) Nash equilibria using the following procedure:

(a) Determine each player's rationalizable strategies $\rho_i^{\infty}(A)$.

(b) Pick a player $i \in \{1,2\}$ (for instance, the one with fewer strategies). For each non empty subset $C_i \subseteq \rho_i^{\infty}(A)$, find all Nash equilibria with $Supp(\alpha_i) = C_i$.

Why is this a good procedure? If α is a Nash equilibrium with $Supp(\alpha_1) = C_1$, then all strategies in C_1 must be optimal. This has implications for α_2. But since α_2 must also be optimal, there are in turn implications for α_1. In the end, one either finds one or more equilibria or reaches a contradiction.

A General Algorithm (formal). Let $[u_i^{k\ell}]$ be the payoff matrix of player i, with generic entry $u_i^{k\ell}$. Player 1 chooses the rows (indexed by k) and player 2 the columns (indexed by ℓ).

Step 1: Eliminate all iteratively dominated actions (by Theorem 5 and Corollary 2 such actions are played with zero probability in equilibrium). The order of elimination is irrelevant.

Step 2: For any pair of non-empty subsets $A_1^* \subseteq A_1$ and $A_2^* \subseteq A_2$, compute the set of mixed equilibria, (α_1, α_2), such that $\text{Supp}\alpha_1 = A_1^*$ and $\text{Supp}\alpha_2 = A_2^*$. This set, which could be empty, is computed as follows (we consider the non trivial case in which the sets contain at least two actions). To simplify the notation, assume that $A_1^* = \{1, ..., m_1^*\}$ and $A_2^* = \{1, ..., m_2^*\}$ and denote by α_i^m the generic probability that action m of player i is played. Solve the following systems of linear equations and inequalities with unknown $\alpha_1 = (\alpha_1^1, ..., \alpha_1^{m_1^*}) \in \Delta(A_1^*)$ and $\alpha_2 = (\alpha_2, ..., \alpha_2^{m_2^*}) \in \Delta(A_2^*)$:

$$\begin{aligned} \sum_{k=1}^{m_1^*} u_2^{k\ell}\alpha_1^k &= \sum_{k=1}^{m_1^*} u_2^{k1}\alpha_1^k,\ \ell = 2, ..., m_2^*, \\ \sum_{k=1}^{m_1^*} u_2^{k\ell}\alpha_1^k &\leq \sum_{k=1}^{m_1^*} u_2^{k1}\alpha_1^k,\ \ell = m_2^*+1, ..., |A_2^*|; \end{aligned} \tag{1}$$

$$\begin{aligned} \sum_{\ell=1}^{m_2^*} u_1^{k\ell}\alpha_2^\ell &= \sum_{\ell=1}^{m_2^*} u_1^{1\ell}\alpha_2^\ell,\ k = 2, ..., m_1^*, \\ \sum_{\ell=1}^{m_2^*} u_1^{k\ell}\alpha_2^\ell &\leq \sum_{\ell=1}^{m_2^*} u_1^{1\ell}\alpha_2^\ell,\ k = m_1^*+1, ..., |A_1^*|. \end{aligned} \tag{2}$$

The subset of equations in (1) determines the set of mixed actions of player 1 that make player 2 indifferent between the actions in subset A_2^*. The subset of inequalities determines the set of mixed actions of player 1 that make the action $a_2 = 1$ (and so *all the actions in* A_2^*) weakly preferred to the actions that do not belong to A_2^*. For any α_1 that satisfies the system (1), player 2 has no incentive to "deviate" from a mixed action with support A_2^*. Similar considerations hold for system (2). Such system determines the set of mixed actions of player 2 that make player 1 indifferent among all the actions in A_1^* and at the same time make such actions weakly preferred to all the others. Therefore, *the indifference conditions for player 1 determine the equilibrium randomization(s) of player 2, the indifference conditions for player 2 determine the equilibrium randomization(s) of player 1.*