ECON 521, Discussion Section 5

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1. Suppose there are three candidates in an election and the set of possible positions they can take is $\{0, 1, 2, \ldots, l\}$, where l is even. Each voter has his own position, and will vote for whichever candidate is closest to his position. If two candidates have the same position, voters closest to that position will split evenly across the two candidates. Each candidate has the option of staying out of the race, which she regards as better than losing and worse than tying for first place. Show that if less than one third of the citizens' favorite positions are equal to the median favorite position (m), then the game has no Nash equilibrium. Argue as follows. First, show that the game has no Nash equilibrium in which a single candidate enters the race. Second, show that in any Nash equilibrium in which more than one candidate enters, all candidates that enter tie for first place. Third, show that there is no Nash equilibrium in which two candidates enter the race. Fourth, show that there is no Nash equilibrium in which all three candidates enter the race and choose the same position. Finally, show that there is no Nash equilibrium in which all three candidates enter the race and do not all choose the same position.

2. Consider the following n-player game. Simultaneously and independently, the players each select either X, Y, or Z. The payoffs are defined as follows. Each player who selects X obtains a payoff equal to γ, where γ is the number of players who select Z. Each player who selects Y obtains a payoff of 2α where α is the number of players who select X. Each player who selects Z obtains a payoff of 3β, where β is the number of players who select Y. Note that $\alpha + \beta + \gamma = n$.
   (a) Suppose $n = 2$. Represent this game in the normal form by drawing the appropriate matrix.
   (b) In the case of $n = 2$, does this game have a NE in pure strategies? Does it have a mixed equilibrium? In either case, find the equilibrium.
   (c) Suppose $n = 11$. Does this game have a NE? If so, describe the equilibrium.

3. Argue that if iterated deletion of strictly dominated strategies eliminates all but one outcome, then (a) that outcome is a Nash equilibrium and (b) there cannot be any other Nash equilibria. (Hint: Use the relationships between rationalizability and iterated deletion of strictly dominated strategies, and Corollary 2 in Chapter 5 of the lecture notes.)

4. Suppose you know the following about a particular two-player game: $A_1 = \{A, B, C\}$, $A_2 = \{X, Y, Z\}$, $u_1(A, X) = 6$, $u_1(A, Y) = 0$, and $u_1(A, Z) = 0$. In addition, suppose you know that the game has a mixed equilibrium in which (a) the players select each of their actions with positive probability, (b) player 1's expected payoff in equilibrium is 4, and (c) player 2's expected payoff in equilibrium is 6. Do you have enough information to calculate the probability that player 2 selects X in equilibrium? If so, what is the probability?

5. Antonio's Spaghetti Factory Ltd. has three creditors. Each creditor is owed d dollars, and the value of the firm is V. The firm is insolvent in that $3d > V$. If just one creditor collects, the firm can pay the debt. If two or more creditors attempt to collect, the firm goes bankrupt (i.e. the firms fail to collect, but still have to pay the collection cost). If one or more creditors collects, the firms value is reduced to v. The costs of collection for each firm is c dollars. The payoff matrix is therefore as follows, where the cell in the i^{th} row and the j^{th} column represents the payoff to a firm who chooses the i^{th} and j other firms chooses to collect.

# others collecting	0	1	2
collect	$d - c$	$v/3 - c$	$v/3 - c$
refrain	$V/3$	$(v - d)/2$	$v/3$

Supposing $d - c > \frac{V}{3}$ and $\frac{v-d}{2} < \frac{v}{3} - c$ what is the NE? Would the three creditors support a law in which they were forbidden to collect from Antonio's Spaghetti Factory Ltd.?

6. Consider the following three player game in which P1 chooses the row, P2 chooses the column, and P3 chooses the matrix.

X	L	R
T	$0, 0, 3$	$0, 0, 0$
B	$1, 0, 0$	$0, 0, 0$

Y	L	R
T	$2, 2, 2$	$0, 0, 0$
B	$0, 0, 0$	$2, 2, 2$

Z	L	R
T	$0, 0, 0$	$0, 0, 0$
B	$0, 1, 0$	$0, 0, 3$

(a) Find the pure strategy NE and list the payoffs for each.

(b) Show that there is a correlated equilibrium in which P3 chooses Y and P1 and P2 play (T, L) and (B, R) with equal probabilities.

(c) Explain the sense in which P3 prefers not to have the information that P1 and P2 use to coordinate their actions.