ECON 521, Discussion Section 7

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1. Alice and Bob are neighbors, and each maintains his/her own garden. Each enjoys looking at the other's garden, as well as his/her own. This enjoyment is increasing in the quality of the gardens, but a higher quality garden requires more effort. Alice has Saturday off from work, while Bob has Sunday off, so Alice works on her garden before Bob. That is, Bob observes the quality of Alice's garden in deciding how much effort to put into his own, but Alice does not observe the quality of Bob's garden before making her decision. For all $i \in A, B$, $u_i(e_i) = e_i(c + e_{-i} - e_i)$, where $c > 0$ is a constant and e_i is i's choice of effort. Find the SPNE effort levels. Is there a first- or second- mover advantage?

   Solution: When Bob makes his decisions, he will best respond to Alice, whatever her effort choice was. That is Bob solves

   $$\max_{e_B} e_B(c + e_A - e_B)$$

   Taking the FOC yields $e_B^* = \frac{c+e_A}{2}$. Alice anticipates this, so she solves

   $$\begin{aligned}&\max_{e_A} e_A(c + e_B - e_A)\\ =&\max_{e_A} e_A(c + \frac{c+e_A}{2} - e_A)\\ =&\max_{e_A} e_A c + e_A\frac{c}{2} - e_A^2\frac{1}{2}\end{aligned}$$

   Taking the FOC yields $e_A^* = \frac{3}{2}c$. That means that $e_B^* = \frac{5}{4}c$. Then $u_A = \frac{3}{2}c\left(c + \frac{5}{4}c - \frac{3}{2}c\right) = \frac{9}{8}c^2$. $u_B = \frac{5}{4}c\left(c + \frac{3}{2}c - \frac{5}{4}c\right) = \frac{25}{16}c^2 > u_A$. Bob has a second-mover advantage as his payoffs are higher than they would be in the static game.

2. Imagine a game in which players 1 and 2 simultaneously and independently select A or B. If they both select A, then the game ends and the payoff vector is $(5, 5)$. If they both select B, then the game ends with the payoff vector $(-1, -1)$. If one of the players chooses A while the other selects B, then the game continues and the players are required to simultaneously and independently select positive numbers. After these decisions, the game ends and each player receives the payoff $(x_1 + x_2)/(1 + x_1 + x_2)$, where x_1 is the positive number chosen by player 1 and x_2 is the positive number chosen by player 2.

   (a) Describe the strategy spaces of the players.

   Solution: $S_i = \{A, B\} \times (0, \infty) \times (0, \infty)$. That is, a strategy must describe a choice between A and B, a positive number for when (A, B) happens and a positive number for when (B, A) happens.

   (b) Does the game have a finite horizon? Is it finite?

   Solution: The game does have a finite horizon because all terminal histories consist of finite sequences of actions. The game is not finite, because there are infinitely many terminal histories.

   (c) Compute the Nash equilibria of this game.

   Solution: Since $(x_1 + x_2)/(1 + x_1 + x_2)$ is between 0 and 1. There can be no NE including (A, B) or (B, A), because the person playing B would have incentive to deviate to A, no matter what the choices of x_1 and x_2.

There can also be no equilibria where they both choose B, because then each would have a profitable deviation to A, since the payoffs from (A, B) and (B, A) are bounded below by zero which is greater than -1.

As for equilibria in which they both play A, obviously there are infinitely many because there exists no profitable deviation no matter what x_1 and x_2 are.

(d) Determine the subgame perfect equilibria.

Solution: There are no subgame perfect equilibria because the subgames following (A, B) and (B, A) have no NE.

3. (Similar to PS2, but dynamic) Imagine there are three major network affiliate television stations in Madison: MadTV1, MadTV2 and MadTV3. All three stations have the option of airing the evening network news program live at 6.00 P.M. or in a delayed broadcast at 7.00 P.M. Each station's objective is to maximize its viewing audience in order to maximize its advertising revenue. The following matrices represent the share of Madison's total population that is "captured" by each station depending on the times at which the news programs are aired. MadTV1 chooses the row, MadTV2 the column and MadTV3 the matrix. For each cell, the first payoff corresponds to MadTV1, the second to MadTV2 and the third to MadTV3. Suppose that MadTV3 chooses its time first. Then MadTV1. Then MadTV2. Note that the order they decide is $TV3$, then $TV1$ then $TV2$. Each observes the choices of the stations made before it.

	6.00	7.00
6.00	$14, 24, 32$	$8, 30, 27$
7.00	$30, 16, 24$	$13, 12, 50$

6.00

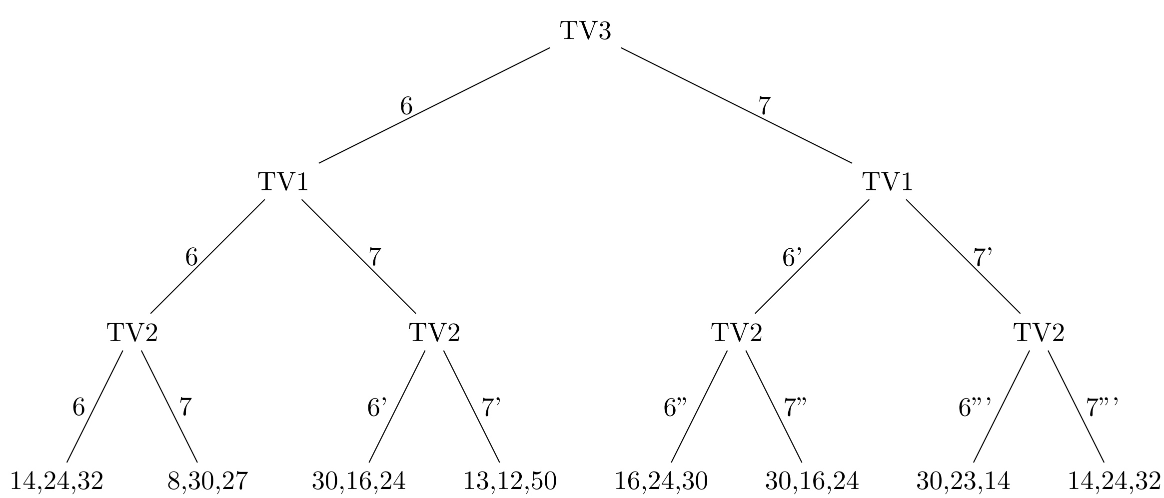
	6.00	7.00
6.00	$16, 24, 30$	$30, 16, 24$
7.00	$30, 23, 14$	$14, 24, 32$

7.00

(a) Draw the game in extensive form.

Solution:

Γ :



(b) Compute the subgame perfect equilibrium.

Solution: By backward induction, SPNE is where $s_2 = 76'6''7'''$, $s_1 = 76'$, $s_3 = 7$. And then the outcome is obviously $(6, 6, 7)$ for TV1, TV2 and TV3 respectively.

(c) Compare your answer with that in PS2, which was a static version of this game.

Solution: The outcome there was $(7, 7, 6)$ for TV1, TV2 and TV3 respectively. The payoffs of $(7, 7, 6)$ are $(12, 12, 50)$, whereas those from $(6, 6, 7)$ are $(16, 24, 30)$. As you can see, TV1 and TV2 are better off and TV3 is worse off when TV3 must act first.

4. A child chooses an action that affects both his own income and his parent's income. He chooses $A > 0$ and incomes for child and parent are

$$I_C(A) = A - A^2 \qquad\qquad I_P(A) = A - \frac{1}{2}A^2$$

What level of action A maximizes the child's income? Call this A_C^*. What level of action maximizes the joint income (i.e. the sum of the two incomes above)? Call this A_J^*.

Now suppose that the child does not care about his parent - he only cares about how much money he has. One the other hand, the parent does care about the child, and may choose to transfer an amount of money B to the child - think of this as a bequest. Their utility functions are as follows:

$$u_C = \sqrt{I_C(A) + B} \qquad\qquad u_P = \sqrt{I_P(A) - B} + \frac{1}{2}u_C$$

The child chooses A before the parent chooses B, and the parent observes A before choosing B. Use backward induction to solve this game. That is, you should first find how the parent best responds to the child's choice of A. Then, given this, you should find what A the child chooses. Once you have found the SPNE through backward induction, comment on how it compares to A_C^* and A_J^*.

Finally, try the problem again except consider the case in which the parent cares very little about his son. That is, replace u_P of above with $u_P' = \sqrt{I_P(A) - B} + \frac{1}{10}u_C$, holding everything else constant. How does this change the child's choice of A in the SPNE?

(Hint: This problem has a fair amount of algebra. And you'll need to use the chain rule in taking derivatives. Life is hard – get used to it.)

Solution: First we must find A_C^*. Just take the derivative of $I_C(A)$, set it equal to zero, and solve to find $A_C^* = \frac{1}{2}$. Doing the same for $I_C(A) + I_P(A)$ gives $A_J^* = \frac{2}{3}$.

Now for the backwards induction. The parent will choose B to best respond to whatever A the child has chosen. They solve the following:

$$\max_B \sqrt{I_P(A) - B} + \frac{1}{2}\sqrt{I_C(A) + B}$$

The FOC is then

$$\begin{aligned} -\frac{1}{2\sqrt{I_P(A) - B^*}} + \frac{1}{4\sqrt{I_C(A) + B^*}} &= 0 \\ 2\sqrt{I_P(A) - B^*} &= 4\sqrt{I_C(A) + B^*} \\ I_P(A) - B^* &= 4(I_C(A) + B^*) \\ B^* &= \frac{I_P(A) - 4I_C(A)}{5} \end{aligned}$$

Given that, the parent is playing B^*, the child solves

$$\begin{aligned}
&\max_A \sqrt{I_C(A) + B^*} \\
=&\max_A \sqrt{A - A^2 + \frac{I_P(A) - 4I_C(A)}{5}} \\
=&\max_A \sqrt{A - A^2 + \frac{A - \frac{1}{2}A^2 - 4(A - A^2)}{5}} \\
=&\max_A \sqrt{A - A^2 - \frac{3}{5}A + \frac{7}{10}A^2} \\
=&\max_A \sqrt{\frac{2}{5}A - \frac{3}{10}A^2}
\end{aligned}$$

We take a first order condition (using the chain rule):

$$\begin{aligned}
\frac{1}{2\sqrt{\frac{2}{5}A^* - \frac{3}{10}A^{*2}}}\left(\frac{2}{5} - \frac{6}{10}A^*\right) &= 0 \\
\frac{2}{5} - \frac{6}{10}A^* &= 0 \\
A^* &= \frac{2}{3}
\end{aligned}$$

This is the neat counter-intuitive result. Even though the child doesn't care about the income of the parent directly, he actually chooses the action that maximizes the joint income of parent and child, rather than just maximizing his own income.

Even more counter-intuitive is that the same thing happens even if the parent cares a lot less about the child (and therefore bequests a lot less). You can repeat the exercise with $u_P' = \sqrt{I_P(A) - B} + \frac{1}{10}u_C$. Do all the same steps and be careful with algebra – and you'll still find that $A^* = \frac{2}{3}$. In fact, this same result holds for any $u_P'' = \sqrt{I_P(A) - B} + ku_C$, so long as $k > 0$. So as long as the parent cares about the child to some extent (which may be arbitrarily small), the SPNE results in the child choosing the action that maximizes joint income. Quite surprising, right? This is the *rotten child problem*, first analyzed by Becker (1974). He proved this result with more general conditions.