ECON 521, Discussion Section 8

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1. Consider a game similar to Cournot except one firm first decides how much to spend on advertising. That is, Firm 1 first chooses a level of advertising a. Then, Firm 1 and 2 simultaneously choose how much to produce. The cost of advertising at level a is $\frac{2a^3}{81}$ and the market price is $p = a - q_1 - q_2$. Assume the good costs nothing to produce. Find the SPE.

   Solution: To find the SPE, we can first find the NE in the subgame that occurs after the advertising decision is made, then we can use backward induction to tackle the advertising decision. P1 and P2 respectively solve the following problems:

$$\max_{q_1}(a - q_1 - q_2)q_1 - \frac{2a^3}{81}$$
$$\max_{q_2}(a - q_1 - q_2)q_2$$

   Taking the derivative and setting it equal to zero for each yields the following first-order conditions

$$a - 2q_1 - q_2 = 0$$
$$a - 2q_2 - q_1 = 0$$

   Solving those simultaneously yields $q_1^* = q_2^* = \frac{a}{3}$. Now we can plug that in to Firm 1's choice of advertising expenditure. The problem is:

$$\max_{q_1}(a - q_1 - q_2)q_1 - \frac{2a^3}{81}$$
$$= \max_{q_1} \frac{a^2}{9} - \frac{2a^3}{81}$$

   Then the FOC is $\frac{2a}{9} - \frac{6a^2}{81} = 0$. Solving gives $a^* = 3$. Therefore the SPE is $\left(\left(3, \frac{a}{3}\right), \frac{a}{3}\right)$. Note that it is technically incorrect to say that the SPE is $((3, 1), 1)$ because playing $(1, 1)$ is only the NE in the subgame if $a = 3$.

2. Consider a two-period game in which one agent is selling up to four units of a good. The good has no cost to the seller (or you can reason that he has already acquired it), but does have value to buyers. There are two types of buyers, high-type and low-type. Their value from owning the good in each period is as follows:

Valuations:

	1	2
H	1200	500
L	500	200

   That is, if a high-type buyer purchases the good in period 1, it provides him total utility of 1700, and he doesn't have to repurchase it in period 2. The seller may charge a different price for the good in each period. Suppose there are two high-type buyers and two low-type buyers. The seller wants to maximize profits, and can vary the prices across the periods but not directly discriminate between the two types of clients.

   (a) How many subgames are there in this game?

   Solution: There are infinitely many, because any price that the seller might set in the first period would induce its own subgame.

(b) What is the maximum profit the seller could make in a pricing scheme in which he sold to everybody in the first period?

Solution: To sell to everybody in the first period, he could charge at most $p_1 = 700$ in the first period. Also, though, the agents would have to prefer that to buying in the second period. So $p_2 \geq 200$. Then his profits are obviously $700 \times 4 = 2800$.

(c) What is the maximum profit the seller could make in a pricing scheme in which he sold to everybody in the second period?

Solution: To sell to everybody in the second period, he could charge at most $p_2 = 200$ in the second period. Also, though, the agents would have to prefer that to buying in the first period. So $p_1 \geq 1400$. Then his profits are obviously $200 \times 4 = 800$.

(d) What is the maximum profit the seller could make in a pricing scheme in which he sold to just the high-types in the first period and didn't sell to the low-types?

Solution: If he was selling to just the high-types in the first period, he could charge $p_1 = 1700$. It would need to be that the high-types weakly prefer to purchase it in the first. Therefore, $p_2 \geq 500$ and profits are $1700 \times 2 = 3400$.

(e) What is the maximum profit the seller could make in a pricing scheme in which he sold to the high-types in the first period and the low-types in the second period?

Solution: If he is selling to the low types in the second period, he maximizes his profit by choosing $p_2 = 200$. As for p_1, note that he cannot set it equal to 1700 because then the high-types prefer to wait and get a surplus of 300 in the second period. Instead, he can charge at most 1400. Total profit here is then $1400 \times 2 + 200 \times 2 = 3200$.

(f) Are there any other possible pricing schemes? Give an intuitive argument to rule them out.

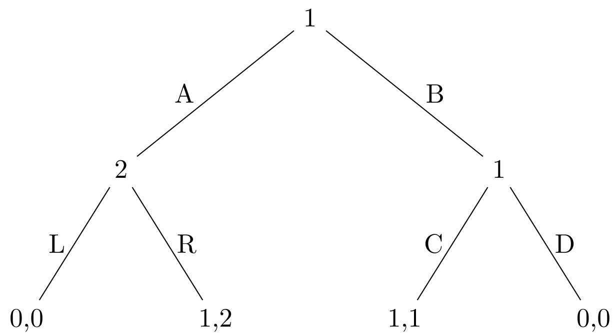
Solution: Sure - he could sell to just the high-types in the second period. Or just the low-types in either period. Or just sell to nobody. Or sell to the high-types in the second period and the low in the first. But in each case, either no such pricing scheme exists or it is clearly suboptimal

(g) Is the maximum profit you have found so far consistent with SPE? If not, then what is the SPE?

Solution: No, selling to just the high-types is not consistent with SPE. The reason is that it required him not to sell to the low-types in period 2 – that is it required $p_2 \geq 500$ where he would actually have incentive to sell to the low-types by setting $p_2 = 200$. But then, of course, he could not sell to the high-types in period 1 for the full 1700. Therefore, the SPE is to sell to the high-types in period 1 with $p_1 = 1400$ and to the low-types in period 2 with $p_2 = 200$, giving him a profit of 3200.

3. Consider the following game (noting that P1 chooses between C and D – that isn't a typo):

$\mathbf{\Gamma}$:



(a) Why does backwards induction not have a solution in this game?

Solution: Backwards induction would imply R and C at the second level of the tree. However, then in deciding between A and B, P1 is indifferent because there is a tie in the payoffs. Therefore, backwards induction has no solution here.

(b) Show the full normal/strategic form of this game.

Solution:

Normal form :

	L	R
AC	0,0	1,2
AD	0,0	1,2
BC	1,1	1,1
BD	0,0	0,0

(c) Show the reduced normal form.

Solution:

Reduced Normal form :

	L	R
A	0,0	1,2
BC	1,1	1,1
BD	0,0	0,0

(d) Find all SPE.

Solution: There are three subgames. In the full game, $(AC, R), (AD, R), (BC, L)$ and (BC, R) are NE. In the left subgame, only R is a best response. And in the right subgame, only C is a best response, so the set of SPE is the set of NE removing those not including R and C. That is, $(AC, R), (BC, R)$ are the SPE.

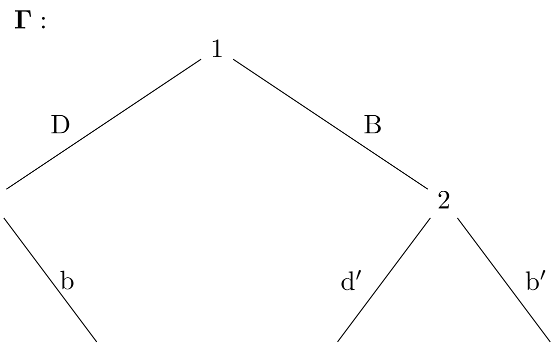
(e) Using the normal form (not the reduced normal form, though it doesn't particularly matter) find an order of Iterated Weak Dominance that eliminates an SPE. Then find an order in which both SPE survive.

Solution: BC weakly dominates AC and AD – you could also use the reduced form and says it weakly dominates A. Since (AC, R) is a SPE, this order eliminates an SPE.

If instead we do it in the order corresponding to backwards induction, i.e. starting from the bottom of the tree, we first delete L because it is weakly dominated by R, then you'll see that $(AC, R), (AD, R)$ and (BC, R) all survive. This set of survivors contains the set of SPE, and the additional survivor (AD, R) is payoff equivalent to the SPE, as we discussed in class.

4. Consider a variation of the burning money game in which P1 has an option to burn money. Then, after observing P1's choice, P2 has the option to burn money. Then the two players play BoS - see the full game below in which D/d stands for *don't burn*, B/b stands for *burn*, F/f stands for *football* and C/c stands for *concert*. Construct the reduced normal form of the game and find the set of outcomes that survive iterated deletion of weakly dominated actions. Compare it with the outcome of the burning money game from the lecture.

   Hint: Try to go straight to the reduced normal form skipping the normal form here. The problem with the non-reduced normal form is that P1 has $2^5 = 32$ *possible strategies, and P2 has* $2^6 = 64$ *possible strategies, so the full normal form is a* 32×64 *matrix. The reduced normal form is still* 8×16*, which still sucks, but is at least doable.*



	f	c
F	3,1	0,0
C	0,0	1,3

	f′	c′
F′	3,0	0,-1
C′	0,-1	1,2

	f″	c″
F″	2,1	-1,0
C″	-1,0	0,3

	f‴	c‴
F‴	2,0	-1,-1
C‴	-1,-1	0,2

Solution: The reduced normal form is as follows. I have broken it into two parts (each with all of P1's strategies but half of P2's) so it fits:

	dd′ff″	dd′fc″	dd′cf″	dd′cc″	db′ff‴	db′fc‴	db′cf‴	db′cc‴
DFF′	3,1	3,1	0,0	0,0	3,1	3,1	0,0	0,0
DFC′	3,1	3,1	0,0	0,0	3,1	3,1	0,0	0,0
DCF′	0,0	0,0	1,3	1,3	0,0	0,0	1,3	1,3
DCC′	0,0	0,0	1,3	1,3	0,0	0,0	1,3	1,3
BF″ F‴	2,1	-1,0	2,1	-1,0	2,0	-1,-1	2,0	-1,-1
BF″ C‴	2,1	-1,0	2,1	-1,0	-1,-1	0,2	-1,-1	0,2
BC″ F‴	-1,0	0,3	-1,0	0,3	2,0	-1,-1	2,0	-1,-1
BC″ C‴	-1,0	0,3	-1,0	0,3	-1,-1	0,2	-1,-1	0,2

	bd′f′f″	bd′f′c″	bd′c′f″	bd′c′c″	bb′f′f‴	bb′f′c‴	bb′c′f‴	bb′c′c‴
DFF′	3,0	3,0	0,-1	0,-1	3,0	3,0	0,-1	0,-1
DFC′	0,-1	0,-1	1,2	1,2	0,-1	0,-1	1,2	1,2
DCF′	3,0	3,0	0,-1	0,-1	3,0	3,0	0,-1	0,-1
DCC′	0,-1	0,-1	1,2	1,2	0,-1	0,-1	1,2	1,2
BF″ F‴	2,1	-1,0	2,1	-1,0	2,0	-1,-1	2,0	-1,-1
BF″ C‴	2,1	-1,0	2,1	-1,0	-1,-1	0,2	-1,-1	0,2
BC″ F‴	-1,0	0,3	-1,0	0,3	2,0	-1,-1	2,0	-1,-1
BC″ C‴	-1,0	0,3	-1,0	0,3	-1,-1	0,2	-1,-1	0,2

Whew. Now, just keep iteratively deleting weakly dominated strategies. If I've done it correctly and you have also done it correctly, you should get down to $(DCC', dd'cc'')$ and $(DCC', db'cc''')$. In either case, the outcome is D, d, C''', c'''

and the payoffs are $(1, 3)$. So the idea is that it doesn't really matter if both players have the option to burn money or just one does. Rather, whoever has the last option (chronologically) to burn money can get their preferred equilibrium (without actually burning the money, of course), given forward induction reasoning.