ECON 521, Discussion Section 9

TA: Shane Auerbach (sauerbach@wisc.edu) ; Date: 10/31/14

1. Evaluate the present value of each of the following infinite sequences given discount rate δ.
   (a) $(1, 1, 1, \ldots)$
   (b) $(x, x, x, \ldots)$
   (c) $(0, 0, x, x, x, \ldots)$
   (d) $(2, 0, 2, 0 \ldots)$
   (e) $(0, 2, 0, 2 \ldots)$
   (f) $(1, 2, 3, 1, 2, 3, \ldots)$

2. Suppose you are considering two possible infinite sequences of payoffs, where d is the sequence resulting from deviating and b is the sequence resulting from behaving:

$$d = (w, x, x, x, \ldots) \qquad\qquad b = (y, z, z, z, \ldots)$$

   Why is it appropriate to say that the deviation is profitable if and only if $(1 - \delta)w + \delta x > (1 - \delta)y + \delta z$?

3. Consider the following prisoner's dilemma game (slightly different payoffs from what he had in class, but it's this way in some textbooks):

	C	D
C	2,2	0,3
D	3,0	1,1

   You could interpret the grim trigger strategy in two ways. For each of the following interpretations, represent the strategy profile as an autonoma and find whether or not it is a SPE (and if so, for what range of δ).
   (a) Each player plays C so long as the other played C in the previous round. If a player plays D, **the other player** plays D in all subsequent rounds.
   (b) Each player plays C so long as the other played C in the previous round. If a player plays D, **both players** play D in all subsequent rounds.

4. Roughly speaking, the folk theorem shown in the lecture applied to the prisoner's dilemma says that no matter how minor the stage-game punishment NE (D, D), and no matter how large the payoff from deviating is, there exists a δ such that that punishment is severe enough to incentivize players to play (C, C) in every period. That is, even if the prisoner's dilemma is as follows,

	C	D
C	2,2	0,999999
D	999999,0	2-ϵ, 2-ϵ

where ϵ is an arbitrarily small positive number, there still exists a δ that's very close to 1 such that grim-trigger (in which the outcome is play they (C, C) in every period) is an SPE.

Make a similar argument in the other direction. That is, in the following game

	C	D
C	2,2	x-1,3
D	3,x-1	x,x

where $x \in \mathbb{R}$, show that for any $\delta > 0$, there exists an x^* such that for all $x < x^*$, there exists a grim trigger SPE in which the outcome is that each player plays C in every period. In words, you're trying to argue that even if a person is very myopic, i.e. barely cares about the future at all, you can still come up with a future punishment severe enough such that they play C. Of course, if $\delta = 0$, this isn't true. Comment on what happens to x^* as $\delta \to 0$ and as $\delta \to 1$.

5. Suppose that you are playing the infinitely-repeated bertrand duopoly - that is, two firms compete on price period after period indefinitely. Each firm's unit cost is constant, equal to c. Denote the total demand for the good at the price p by $D(p)$. Let $\pi(p) = (p - c)D(p)$ for very price p, and assume that D is such that the function π is continuous and has a single maximizer, denoted by p^m, i.e. the monopoly price.

   (a) Let s_i be the strategy of firm i in the infinitely-repeated game of this strategic game that charges p^m in the first period and subsequently as long as the other firm continues to charge p^m and punishes any deviation from p^m by the other firm by choosing the price c for k periods, then reverting to p^m. Given any value of δ, for what values of k is the strategy pair (s_1, s_2) a NE?

   (b) Let s_i be the following strategy for firm i in the infinitely repeated game:

   - in the first period charge the price p^m
   - in every subsequent period, charge the lowest of all the prices charged by the other firm in all previous periods.

   That is, firm i matches the other firm's lowest price. Is the strategy pair (s_1, s_2) a NE of the infinitely repeated game for any discount factor less than 1?