Monopoly and price discrimination

Econ 301 Handout solutions

April 19, 2012

1 Basic Monopoly

1. Set MR = MC to get the profit maximizing output choice. Here, MC is the partial derivative of TC with respect to output. Thus we have y = 300 - 2y. This means profit is maximized at an output of 100 units and a price (given by the demand curve at that output level) of $200.

2. Prudcer surplus is the area above the supply curve and below the market price. This is two areas. One is the box between prices of $200 and $100 on the vertical axis and between 0 and 100 units of the horizontal axis. The second area is the triangle between $0 and $100 on the vertical axis and 0 and 100 units on the horizontal axis. In total this is $15,000.

3. Consumer surplus is the area above the market price and below the demand curve. This totals $5,000.

4. The monopoly outcome is not pareto efficient, because, most notably, there is a deadweight loss ($2,500).

2 Monopoly and price discrimination

Airline BestFly is a monopolist in the market of Island Zee; it has to decide how many tickets are to be sold for the next holiday season. The fixed costs of running an airline are $F = \$40,000$, and the variable cost is given by $c(y) = 100y$. BestFly faces inverse demand curve $p(y) = 600 - y$.

1. Suppose that BestFly can perfectly discriminate (1st degree price discrimination). What is the profit it can earn in such case?
   Solutions: In this case monopolist profit equals the total gains to trade, consumer surplus is zero:

   The monopolist produces so that the price for the last unit equals to MC. $p(y) = MC(y) \Longrightarrow 600 - y = 100 \Longrightarrow y = 500$.

   $\pi = PS = \frac{1}{2} * 500 * 500 - 40,000 = 125,000 - 40,000 = 85,000$

   $CS = 0$.

2. There are two groups of travelers with different inverse demand curves: business travelers, $p^B = 1200 - 4y^B$; and tourists, $p^T = 400 - \frac{4}{3}y^T$. Show that if BestFly does not discriminate, the aggregate inverse demand is the same as in the first part of the problem.

   Solutions: The demand functions for the two groups are $y^B = 300 - \frac{1}{4}p^B$ and $y^T = 300 - \frac{3}{4}p^T$. The aggregate demand is obtained by adding individual demand functions: $y(p) = y^B(p) + y^T(p) = 300 - \frac{1}{4}p + 300 - \frac{3}{4}p = 600 - p$, which gives the same inverse demand function ($p(y) = 600 - y$).

3. Find the level of sales, prices, profits and elasticity of demand for each of the market segments under 3rd degree price discrimination.

   Solutions: Consider separately each market segment. On the business segment, the $TR^B = y^B(1200 - 4y^B)$, therefore$MR^B = 1200 - 8y^B$.

   On the tourist segment, the $TR^T = y^T(400 - \frac{4}{3}y^T)$, therefore$MR^T = 400 - \frac{8}{3}y^T$.

   In this case, the monopolist produces so that $MR^B(y^B) = MR^T(y^T) = MC(y^B + y^T)$. On the business segment,

$$MR^B(y^B) = 1200 - 8y^B = 100$$
$$y^B = 137.5$$

$$p^B(y^B = 137.5) = 1200 - 4 * 137.5 = 650$$

$$\pi^B = 137.5 * (650 - 100) = 75,625$$

$$\epsilon^B = y^B(p^B)\frac{p^B}{y^B} = -\frac{1}{4} * \frac{650}{137.5} = -1.18$$

   On the tourist segment,

$$MR^T(y^T) = 400 - \frac{8}{3}y^T = 100$$
$$y^T = 112.5$$

$$p^T(y^T = 112.5) = 400 - \frac{4}{3} * 112.5 = 250$$

$$\pi^T = 112.5 * (250 - 100) = 16,875$$

$$\epsilon^T = y^T(p^T)\frac{p^T}{y^T} = -\frac{3}{4} * \frac{250}{112.5} = -1.67$$

   The total profit of the monopolist is $\pi = PS = \pi^B + \pi^T - F = 75,625 + 16,875 - 40,000 = 52,500$.

   The consumer surplus is $\frac{1}{2} * (1200 - 650) * 137.5 + \frac{1}{2} * (400 - 250) * 112.5 = \frac{1}{2} * 550 * 137.5 + \frac{1}{2} * 150 * 112.5 = 37,812.5 + 8437.5 = 46,250$

4. Compare producer and consumer surplus in three cases: uniform price (nondiscriminating monopolist), perfect discrimination and 3rd-degree price discrimination.

   Solutions: We only need to calculate the PS and CS in the uniform price case. In this case, the total revenue of the monopolist is given by $TR = y(600 - y)$. Thus the marginal revenue is $MR = 600 - 2y$. The monopolist produces so that $MR = MC$:

SEE ADDENDUM TO SOLUTION TO THIS PROBLEM ON NEXT PAGE

$$MR = 600 - 2y = 100 = MC$$

$$y = 250$$

$$p = 600 - 250 = 350$$

$$\pi = PS = py - 100y - F = 250 * 350 - 250 * 100 - 40,000 = 22,500$$

$$CS = \frac{1}{2}(600 - 350) * 250 = 31,250$$

3 Demand elasticity

Suppose the demand facing the monopolistic firm is given by

$$y(p) = 2 - p$$

1. Calculate the value of the demand elasticity for $y = 0$, $y = 1$ and $y = 2$.

   Solutions: The elasticity function is $\epsilon(y) = \frac{dy}{dp}\frac{p}{y} = y'(p)\frac{p}{y} = (-1)\frac{2-y}{y}$.

   $\epsilon(0) = (-1)\frac{2-0}{0} = -\infty$,$\epsilon(1) = (-1)\frac{2-1}{1} = -1$, and $\epsilon(2) = (-1)\frac{2-2}{2} = 0$.

2. Write down MR in terms of elasticity ϵ and price p. What is the value of ϵ when $MR = 0$? What about $MR > 0$?

   Solutions: $MR = TR' = (p(y)y)' = p(y) + p'(y)y = p(y)[1 + \frac{p'(y)y}{p(y)}] = p(y)[1 + \frac{1}{\frac{dy}{dp}\frac{p}{y}}]$.

   Note that by definition $\epsilon(p) = \frac{dy}{dp}\frac{p}{y}$. Plugging it into MR function above yields $MR = p[1 + \frac{1}{\epsilon(p)}]$.

   $MR = p[1 + \frac{1}{\epsilon(p)}] = 0 \Longrightarrow p[1 + \frac{1}{\epsilon(p)}] = 0 \Longrightarrow 1 + \frac{1}{\epsilon(p)} = 0 \Longrightarrow \epsilon(p) = -1$.

   $MR = p[1 + \frac{1}{\epsilon(p)}] > 0 \Longrightarrow p[1 + \frac{1}{\epsilon(p)}] > 0 \Longrightarrow 1 + \frac{1}{\epsilon(p)} > 0 \Longrightarrow \epsilon(p) < -1$.

3. Find the markup over a marginal cost MC.

   Solutions: The monopolist always produces so that $MR = MC$. Applying the result from part 2), we have $MC = MR = p[1 + \frac{1}{\epsilon(p)}]$. Or $\frac{MC}{[1+\frac{1}{\epsilon(p)}]} = p$. The markup over marginal cost is $\frac{1}{[1+\frac{1}{\epsilon(p)}]}$.

Addendum to Solutions to Handout 10, Question 2

When we solve question 2, part d, we get the following answers:

$$\begin{array}{lll} PS^{Uniform} = 22500 & PS^{3rdDegree} = 52500 & PS^{Perfect} = 85000 \\ CS^{Uniform} = 31250 & CS^{3rdDegree} = 46250 & CS^{Perfect} = 0 \\ TS^{Uniform} = 53750 & TS^{3rdDegree} = 98750 & TS^{Perfect} = 85000 \end{array}$$

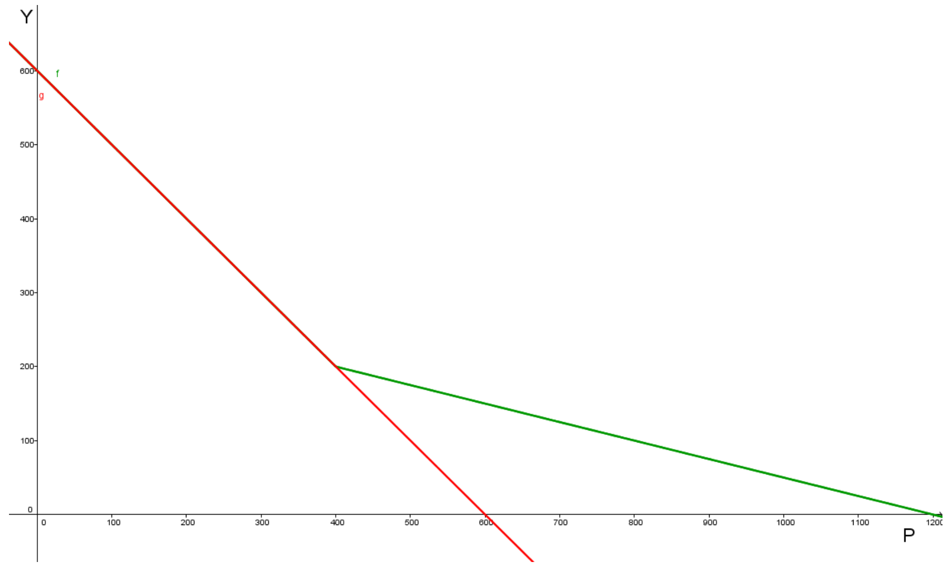
Hopefully something in these numbers surprises you. We know that there is no deadweight loss when the monopolist is able to do perfect price discrimination, but that there is deadweight loss when the monopolist can only do third-degree price discrimination. Therefore, it follows that $TS^{Perfect} \geq TS^{3rdDegree}$, but that isn't the case here. Why?

The answer is actually that the question is not well structured. When you solved for the perfect price discrimation case in part a), you were asked to do so with the demand function $y(p) = 600 - p$ (they actually gave you the inverse demand function, $p(y) = 600 - y$, but you can obviously solve that to get $y(p) = 600 - p$). Then in part b), they split up the market into tourists and businessmen and asked you to aggregate the two to check that they summed to the original $y(p) = 600 - p$. They do of course sum to $600 - p$, however these demands are not actually the same. Here are the two demand functions:

$$\begin{array}{l} \text{Demand 1} : y(p) = 600 - p \\ \text{Demand 2} : y^B(p) = 300 - p^B/4 \text{ and } y^T(p) = 300 - 3p^T/4 \end{array}$$

Note that with the first demand function, if the price is greater than 600, say 700, then the demand is 0 - it actually spits out a negative number, but we don't allow negative demand, so we say it's zero. With the second demand functions, at a price of 700, the tourist demand will be zero, but there will still be positive businessman demand, so the total demand is positive. Therefore, this is an example of how the two demands are different.

Where we went wrong was when we just summed the two demands, over some ranges of prices (in particular greater than 400) we were actually adding a negative demand for the tourists to a positive demand for the businessment, where we should have been just taking the businessman's positive demand as the total demand. Graphically (see the plot below), when we mechanically summed the two demands in b), we got the red line g below, whereas those demand functions, aggregated, are better represented by the green f line below. Note that the green line is under the red line for $p < 400$.



So what is the true producer surplus (which equals total surplus) when we have perfect price discrimination and $y^B(p) = 300 - p^B/4$ and $y^T(p) = 300 - 3p^T/4$? To calculate this properly, just calculate it for each of the two markets separately and sum it.

$$\begin{aligned} PS^{Perfect}_{Total} &= PS^{Perfect}_{B} + PS^{Perfect}_{T} \\ &= \frac{(1200-100)(275)}{2} + \frac{(400-100)(225)}{2} = 185000 \end{aligned}$$

Obviously, $185000 > 98750$, so we do now get the desired result that $TS^{3rdDegree} < TS^{Perfect}$.