Public goods

Econ 301 Handout Solution

April 28, 2011

1. The profit function of Beth's firm given $t^A = 0.25$ is

$$\pi^B = \ln(0.25 + t^B) + \ln(m^B) - t^B - m^B$$

The first order condition for profit maximization yields

$$\frac{1}{0.25 + t^B} = 1$$
$$t^B = 1 - 0.25 = 0.75$$

2. The first order condition for maximization of Beth's profit is

$$\frac{1}{t^A + t^B} = 1,$$

therefore the best response function for firm B is

$$t^B = \begin{cases} 1 - t^A & \text{if } t^A < 1, \\ 0 & \text{if } t^A \geq 1. \end{cases}$$

3. The first order condition for profit maximixation by firm A is

$$\frac{5}{t^A + t^B} = 1,$$

and its best response function is

$$t^A = \begin{cases} 5 - t^B & \text{if } t^B < 5, \\ 0 & \text{if } t^B \geq 5. \end{cases}$$

4. The Nash equilibrium corresponds to the point where the two best response functions intersect, in this case $t^A = 5, t^B = 0$. The total amount of money invested in tsunami alert systems is $t^A + t^B = 5$.

5. Yes, firm B is free riding because its customers value the pubic good less.

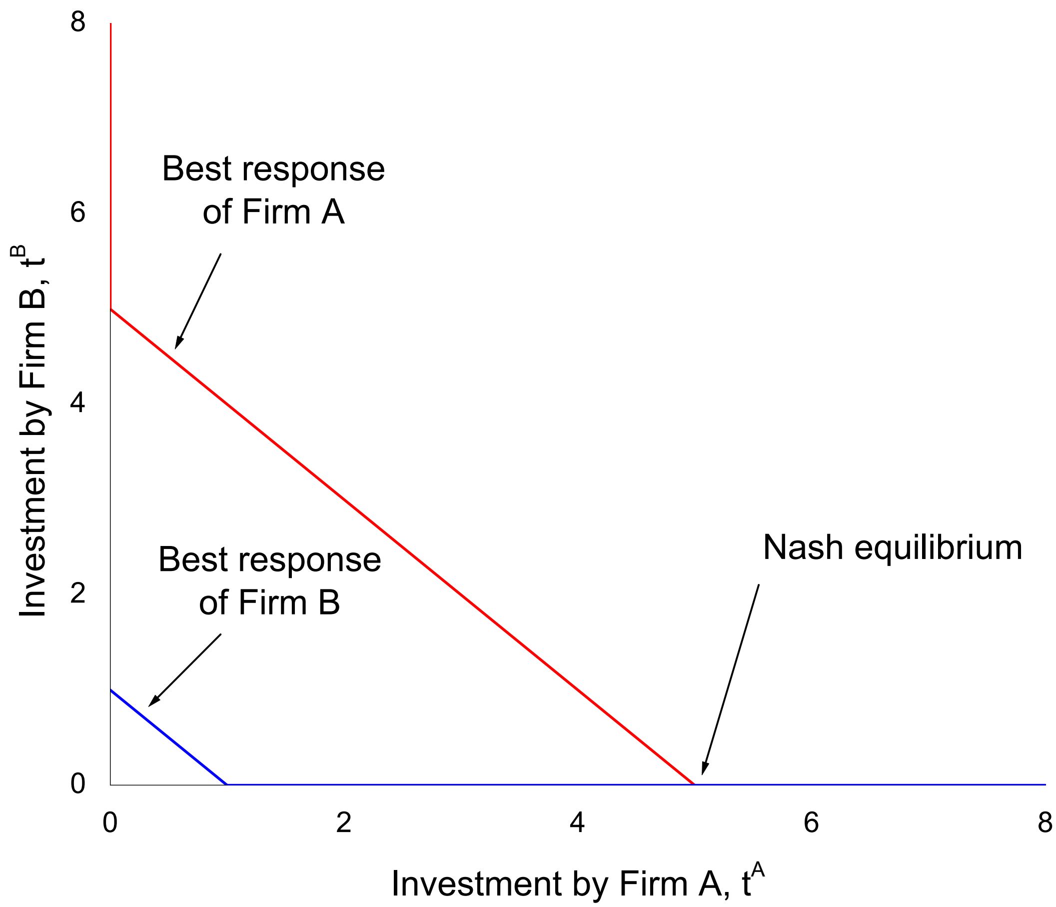
6. To get Pareto efficient joint level of investment in tsunami alert systems, maximize the joint profits of both firms (with $t = t^A + t^B$):

$$\pi = \pi^A + \pi^B = 5 \ln t + \ln m^A - m^A + \ln t + \ln m^B - m^B - t$$

The first order condition for the optimal amount of public good is

$$\frac{6}{t} = 1$$
$$t = 6,$$

greater than the one observed in the market.



3 Production Externalities

Firm S produces steel(s), and also produces a certain amount of pollution(x) which it dumps into a river. Firm F, a fishery, is located downstream and is adversely affected by S's pollution. Suppose that the steel firm's cost function is given by

$$TC_s(s,x) = \frac{1}{4}s^2 - 2x + x^2,$$

and the fishery's cost function is

$$TC_F(f,x) = \frac{1}{2}f^2 + xf.$$

The price of steel and fish is $p_s = 1$ and $p_f = 2$, respectively.

(a) Find the amount of steel s and the amount of pollution x that maximizes the firm's profit.

ANSWER $x = 1$ is immediate from the cost function. At $x = 1$, the steel firm's cost would be minimized. To find the optimal s, observe that the marginal revenue is 1 but the marginal cost is $\frac{1}{2}s$. The optimal s will be 2 because the marginal revenue is equal to the marginal cost at $s = 2$.

(b) Given the amount of pollution x, find the optimal level of production of fish f, and find the fishery's profit.

ANSWER Given x, the marginal cost function boils down to

$$\frac{\partial TC_F}{\partial f} = f + x$$

Setting it to be equal to marginal revenue 2,

$$f + x = 2 \rightarrow f = 2 - x.$$

Observe that the amount of fish f decreases in x. Therefore, the pollution x has *negative* external effect on production of the fishery.

(c) Express the joint profit function.

ANSWER The profit function of the steel firm is

$$\pi_s = p_s s - TC_s = s - \frac{1}{4}s^2 + 2x - x^2 \tag{-3}$$

And the profit function of the fishery is

$$\pi_f = p_f f - TC_f = 2f - \frac{1}{2}f^2 - xf \tag{-2}$$

The joint profit function is obtained by simply adding the two profit functions above:

$$\Pi = \pi_s + \pi_f = \left(s - \frac{1}{4}s^2 + 2x - x^2\right) + \left(2f - \frac{1}{2}f^2 - xf\right) \tag{-1}$$

(d) Find the Pareto efficient level of production (s, x, f).

ANSWER The Pareto efficient production indicates (s, x, f) that maximizes the joint profit function in the sense that it is the most socially desirable. To figure it out, we (partially) differentiate Π we got in part (c) with respect to s, x, and f. It gives us

$$\frac{\partial \Pi}{\partial s} = 1 - \frac{1}{2}s = 0.$$

$$\frac{\partial \Pi}{\partial x} = 2 - 2x - f = 0.$$
$$\frac{\partial \Pi}{\partial f} = 2 - f - x = 0.$$

The Pareto efficient s immediately comes from the first equation; $s = 2$. And the Pareto efficient x and f come from the last two equations: $f = 2$ and $x = 0$.

Problem 3: Adverse Selection

Denote G = Good drum set, B = Bad drum set

1. Prob(G) = Prob(B) = 0.5 ➔ Gains-to-trade = 0.5(60 – 50) + 0.5(20 – 10) = $10
2. For a Good drum set: p_G = (50+60)/2 = $55
   For a Bad drum set: p_B = (20+10)/2 = $15
3. To have a pooling equilibrium, we need the maximum price that a buyer would pay to be greater than the valuation of a seller for a Good drum set.
   The maximum price a buyer would pay for a drum set is equal to his expected valuation of a drum set:
   EV_B = Prob(B)*20 + [1 – Prob(B)]*60
   For a pooling equilibrium, we need $EV_B \geq 50$
   ➔ 20Prob(B) + 60 – 60Prob(B) ≥ 50
   ➔ 40Prob(B) ≤ 10
   ➔ Prob(B) ≤ ¼
   If the probability of encountering a Bad drum set is less than ¼, there would be a pooling equilibrium with both types of drum sets being sold.
4. If Prob(B) = ½ > ¼, the equilibrium will be separating and thus not Pareto efficient (lost gains of trade).
   If Prob(B) = 1/5 < ¼, the equilibrium would be pooling and thus Pareto efficient.

Problem 4: Signaling

1. The amount you would pay under no signaling is equal to the expected productivity:
   (1/3)*6 + (2/3)*18 = $14
2. If a Slow candidate succeeds in making you think he is a Fast candidate, he would gain the difference in wages that you would pay the two types, i.e., GAIN = $18 - $6 = $12
   So, to find the minimum number of tasks that would be a credible signal, n*, we need to find the number of tasks that would cost a Slow type exactly $12. That is:
   $$3n^*/5 = 12$$
   $$n^* = 20 \text{ tasks}$$