Econ 301: Intermediate Microeconomics Spring 2013

Discussion Section #2 Handout January 31st

Concepts Review

A. Marginal Rate of Substitution (MRS): the rate at which you are willing to exchange one good for the other. This rate typically depends upon how many of each you have.

$$MRS_{xy} = -\frac{MU_x}{MU_y} = slope\ of\ the\ IC$$

B. Conditions for Optimal Consumption: If we have n goods we need n conditions to determine optimal consumption. One will come from the budget constraint and the rest $n-1$ typically come from the first derivative of the utility function.

- Budget Constraint: As long as there is at least one "good" the budget constraint will hold with equality because spending more money on that good will increase utility.
- Well-Behaved Preferences (Monotonic and Convex) optimality condition:

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

Intuition: The rate at which you are willing to exchange the goods (the MRS) is the same as the rate at which you can exchange them (the price ratio).

Equivalently, the optimality condition can be rewritten as:

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

Intuition: The marginal utility per dollar is the same for all goods. Otherwise you could take the last dollar you spent on one good, instead spend it on the other good, and make yourself better off.

C. Special Cases

- Perfect Complements
  Problem: The MRS is undefined at the optimal bundle (the kink point)
  Solution: The optimal bundle is always at the kink.
- Perfect Substitutes
  Problem: The MRS is -1 (mostly) everywhere. Thus it may never equal the price ratio, or may equal the price ratio everywhere.
  Solution: Consume only the cheapest good, or choose any bundle on the budget line if the prices are the same.
- Non-convex Preferences (e.g. $U(x,y) = x^2 + y^2$)
  Problem: The first derivative of a function identifies critical points, but some may be minima or inflection points rather than maxima.
  Solution: Compare all interior and corner solutions to see which delivers the highest utility

Practice Problems

Problem 1: Multivariate Calculus Review

Find the first partial derivative with respect to good x_1 and x_2.

a) $f(x_1, x_2) = 5x_1 + 3x_2$
b) $f(x_1, x_2) = \sqrt{x_1 x_2}$
c) $f(x_1, x_2) = \log x_1 + \log x_2$

Problem 2: Cobb-Douglas Functions

Peter's utility function for good x_1 and x_2 is given by $U(x_1, x_2) = x_1^4 x_2$. His current income is \$100 and the prices of x_1 and x_2 are \$20 and \$10, respectively.

a) Express his marginal rate of substitution between x_1 and x_2.
b) Plot the indifference curve that passes through bundle $(1,1)$. Find the MRS at this bundle and depict it on the graph.
c) Write down the two conditions for the optimal choice.
d) Find the optimal bundle for him.

Problem 3: Perfect Complements [2010 Spring Midterm 1]

Michael always consumes three hamburgers (x_1) along with one Coke (x_2). This is the only healthy combination of the two products!

a) Propose Michael's utility function $U(x_1, x_2)$ that represents his preferences over hamburgers and Coke.
b) In the commodity space (x_1, x_2), carefully depict his indifference curve and mark the optimal proportion line.
c) Write down the "two secrets of happiness" that determine his optimal choice. Explain the economic intuition behind the conditions (one sentence for each secret).
d) Find his optimal choice of x_1 and x_1 as a function of p_1, p_2, m. Is the choice interior for any price and income?

Problem 4: Perfect Substitutes

Isobel considers peaches and nectarines to be perfect substitutes. She spends \$5 on these fruits every week. The price of a peach is \$1 a pound and the price of a nectarine is \$2 a pound.

a) Graph two indifference curves - one passing through the bundle $(1,2)$ and the other passing through $(2,2)$.
b) Provide a utility function that represents her preferences. What is the MRS?
c) Find the optimal bundle and mark it on the graph. Is it interior?
d) Suppose that the price of a peach increases to \$3 due to a very dry season. How does it affect Isobel's optimal bundle?

Problem 5: Jeremy's Flowers [2011 Final A]

Jeremy's favorite flowers are tulips (x_1) and daffodils (x_2). Suppose $p_1 = 2, p_2 = 4$ and $m = 40$.

a) Write down Jeremy's budget constraint (a formula) and plot all of Jeremy's affordable bundles in the graph (his budget set). Find the slope of the budget line (a number). Give an economic interpretation for the slope of the budget line (a sentence).
b) Jeremy's utility function is given by $U(x_1, x_2) = \sqrt{(\ln x_1 + \ln x_2)^2 + 7}$. Propose a simpler utility function that represents the same preferences (give a formula). Explain why your utility represents the same preferences (one sentence).
c) Plot Jeremy's indifference curve map (graph), find MRS analytically (give a formula), and find its value at bundle (2,4) (a number). Give economic interpretation of this number (one sentence). Mark its value in the graph.
d) Write down "two secrets of happiness" (two equalities) that allow determining the optimal bundle. Provide their geometric interpretation (one sentence for each). Find the optimal bundle (x_1, x_2). Is your solution interior?
e) Hard: find the optimal bundle given $p_1 = 2, p_2 = 4$ and $m = 40$ and $U(x_1, x_2) = 2x_1 + 3x_2$. Is the solution interior?

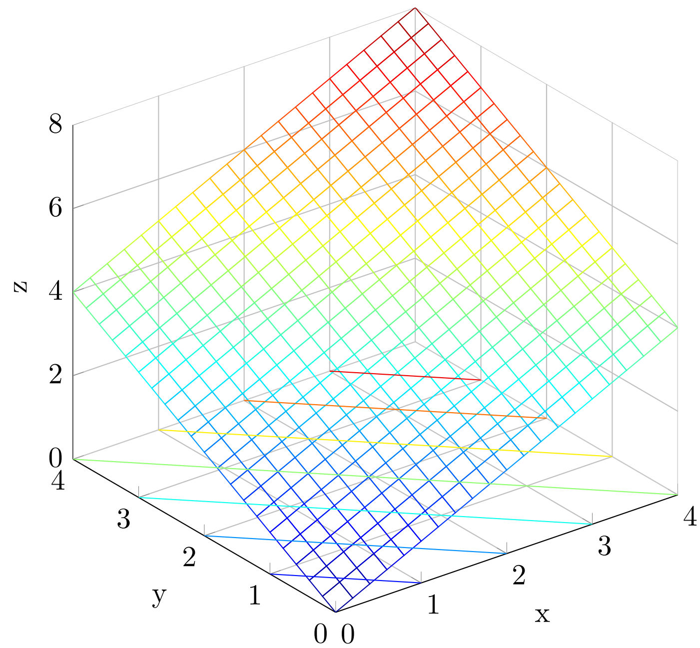

Figure 1: $z = x + y$, i.e. Perfect Substitutes

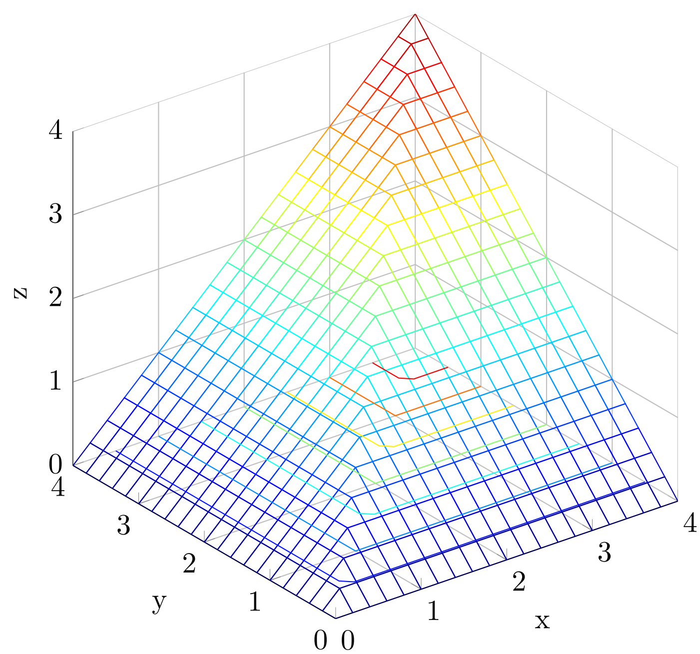

Figure 2: $z = \min(x, y)$, i.e. Perfect Complements

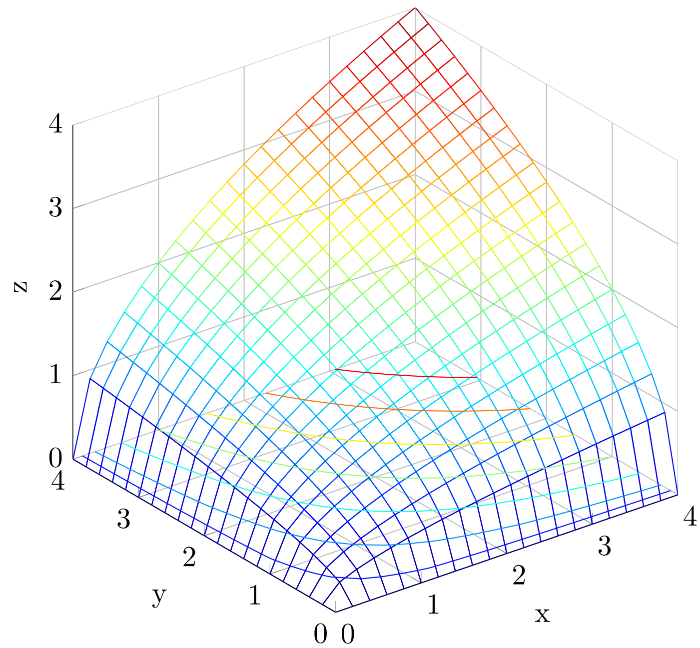

Figure 3: $z = x^{1/2} \cdot y^{1/2}$, i.e. Cobb-Douglas

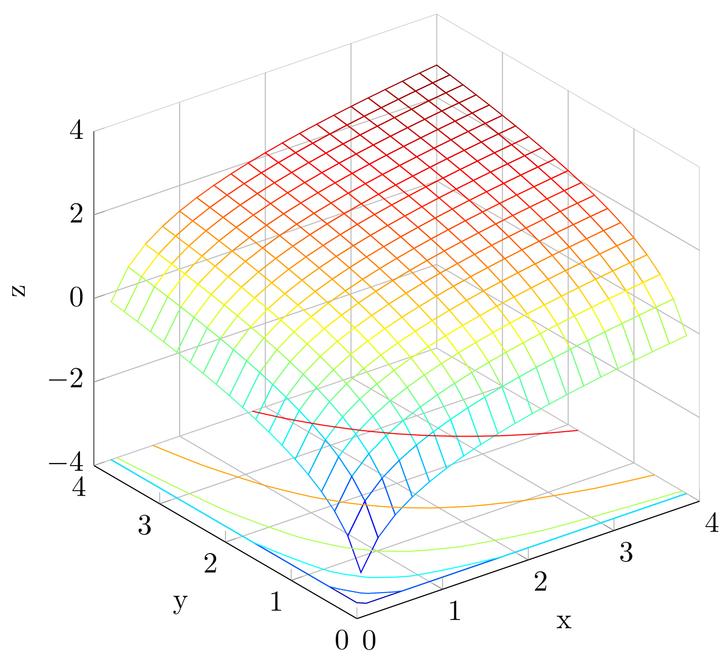

Figure 4: $z = \log(x \cdot y)$, i.e. Logarithmic Utility