

# Demand and Comparative Statics

Econ 301: Week 3 Solutions

February 4, 2011

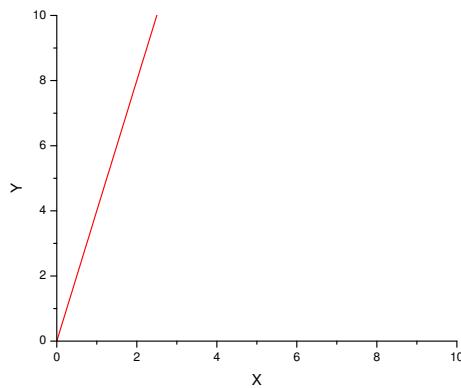
## 1 Example 1: perfect complements

Utility function:  $U(x,y) = \min(4x,y)$ , budget constraint:  $2x + y = 12$

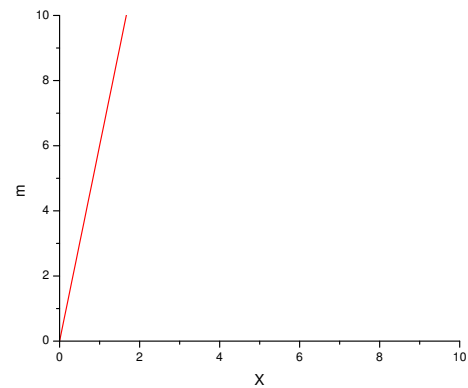
1. Example from one of the sections: cars ( $x$ ) and tires ( $y$ )
2. Use the shortcut formulae from the handout:

$$x = b \frac{m}{p_1 b + p_2 a} = 1 \frac{12}{2 + 4} = 2$$
$$y = a \frac{m}{p_1 b + p_2 a} = 4 \frac{12}{2 + 4} = 8$$

3. Income offer curve coincides with optimal proportion line,  $y = 4x$ ; Engel curve is  $m = 6x$ :



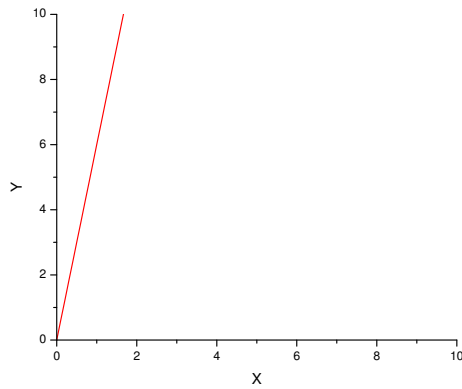
Income offer curve



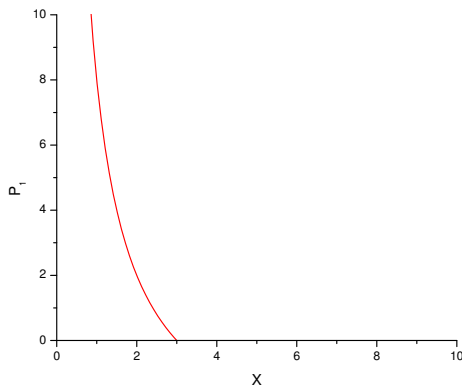
Engel curve

Engel curve is positively sloped, so  $x$  is a normal good.

4. Price offer curve coincides with optimal proportion line,  $y = 4x$ :



5. Demand curve is  $x = \frac{12}{p_1+4}$ :



Demand curve has a negative slope, so this is an ordinary good.

6. Demand for each good decreases as the price of another good increases, so these goods are gross complements.

## 2 Example 2: perfect substitutes

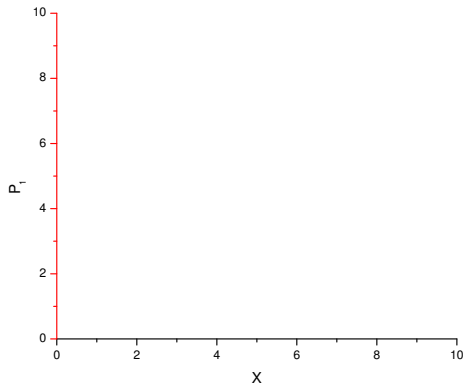
Utility function:  $U(x,y) = x + 4y$ , budget constraint:  $2x + y = 12$

1. Example from one of the sections: mini bagels ( $x$ ) and bread ( $y$ )
2. Use the shortcut formulae from the handout:

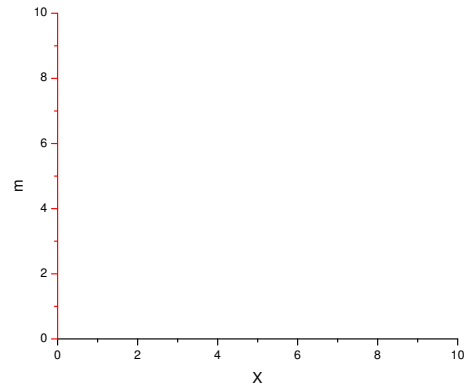
$$\frac{a}{b} = \frac{1}{4} < \frac{p_1}{p_2} = 2 \Rightarrow x = 0, y = 12$$

(corner solution, consume only bread)

3. Both income offer curve and Engel curve are represented by vertical axis:



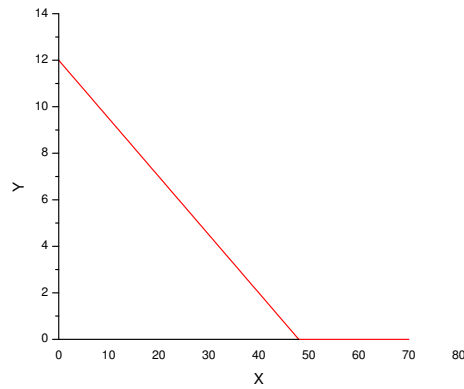
Income offer curve



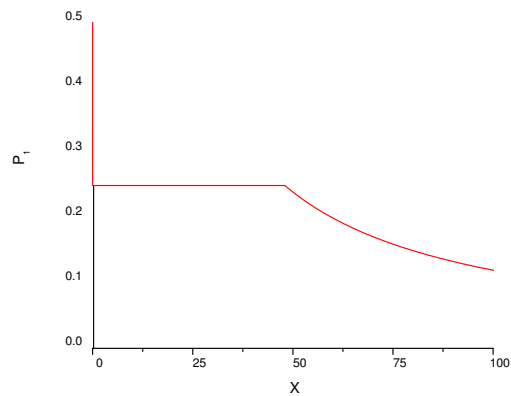
Engel curve

The demand for  $x$  doesn't decrease in income, so  $x$  is a normal good.

4. Price offer curve consists of two segments,  $y = 12 - 1/4x$  if  $0 \leq x \leq 48$ , and  $y = 0$  if  $x \geq 48$ :



5. Demand curve consists of three segments:  $x = 0$  if  $p_1 \geq \frac{1}{4}$ ,  $x$  is between 0 and 48 if  $p_1 = \frac{1}{4}$ ,  $x = \frac{12}{p_1}$  if  $p_1 \leq \frac{1}{4}$ :



As price increases, less of good  $x$  is demanded, so this is an ordinary good.

6. Demand for each good increases as the price of another good increases, so these goods are gross substitutes.

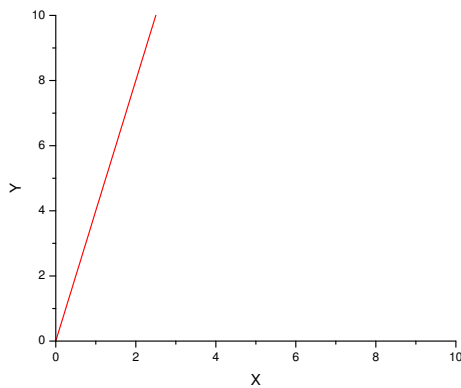
### 3 Example 2: Cobb-Douglas preferences

Utility function:  $U(x,y) = xy^2$ , budget constraint:  $2x + y = 12$

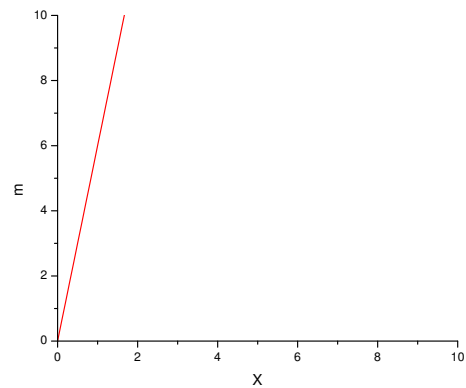
1. Tires and bagels...
2. Use the shortcut formulae from the handout:

$$x = \frac{a}{a+b} \frac{m}{p_1} = \frac{1}{3} \frac{12}{2} = 2, \quad x = \frac{b}{a+b} \frac{m}{p_2} = \frac{2}{3} \frac{12}{3} = 8$$

3. Income offer curve is a line described by equation  $y = 4x$ , Engel curve is a line described by equation  $m = 6x$ :



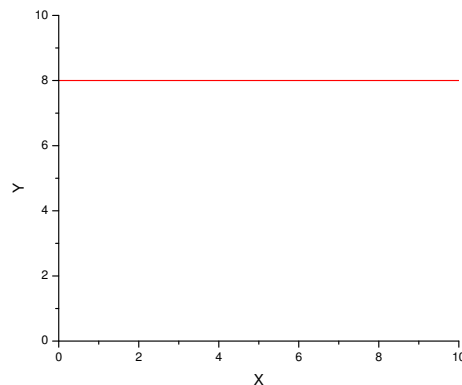
Income offer curve



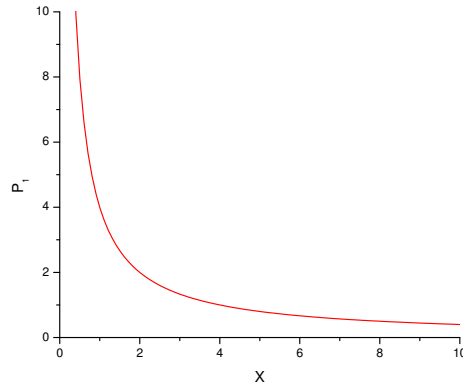
Engel curve

The demand for  $x$  doesn't decrease in income, so  $x$  is a normal good.

4. Price offer curve is a horizontal line,  $y = 8$ :



5. Demand curve is:  $x = 4/p_1$ :



As price increases, less of good  $x$  is demanded, so this is an ordinary good.

6. Demand for each good doesn't depend on the price of another good, so these goods are unrelated (neither gross substitutes, nor gross complements).

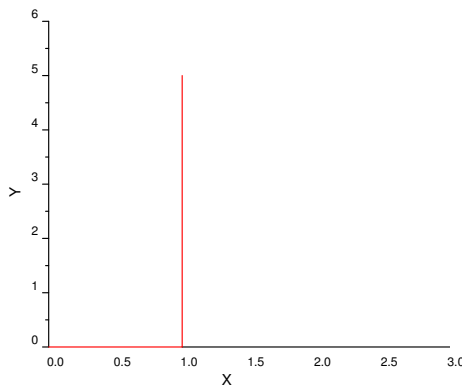
#### 4 Example 2: quasilinear preferences

Utility function:  $U(x, y) = 2\ln x + y$ , budget constraint:  $2x + y = 12$

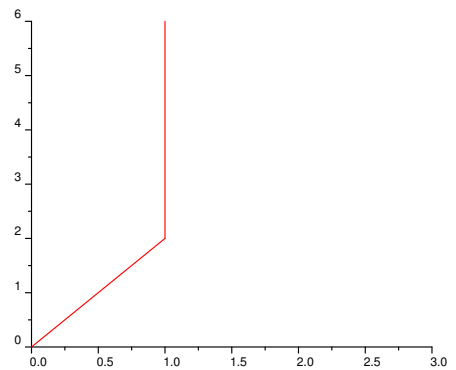
1. Varian: pencils ( $x$ ) and all other goods ( $y$ )
2. We have interior solution, so equate MRS and the slope of budget constraint to get:

$$x = \frac{2 \cdot 1}{2}, y = 12 - 2 \cdot 1 = 10$$

3. Income offer curve is a vertical line at  $x = 1$  and a horizontal line connecting the origin and a point  $(1, 0)$ . Engel curve consists of two segments,  $m = 2x$  if  $0 \leq m \leq 2$  and  $x = 1$  if  $m \geq 2$ :



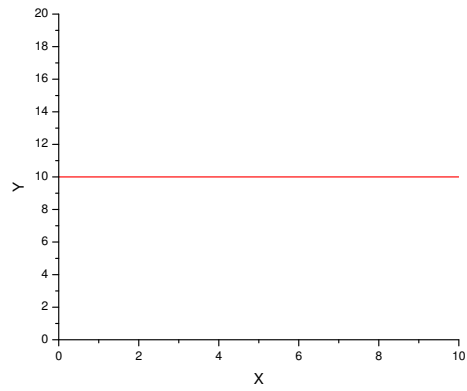
Income offer curve



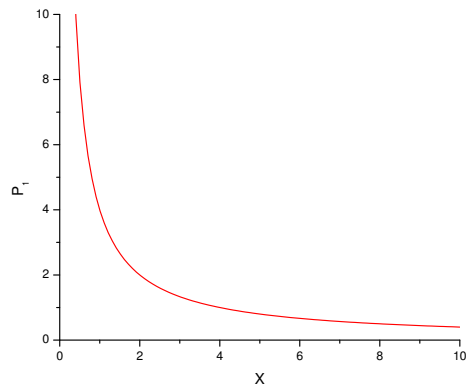
Engel curve

The demand for  $x$  doesn't decrease in income, so  $x$  is a normal good.

4. Price offer curve is a horizontal line,  $y = 10$ :



5. Demand curve (having interior solution) is:  $x = 2/p_1$ :



As price increases, less of good  $x$  is demanded, so this is an ordinary good.