1 Substitution Effect vs. Income Effect

1.1 Cobb-Douglas preferences

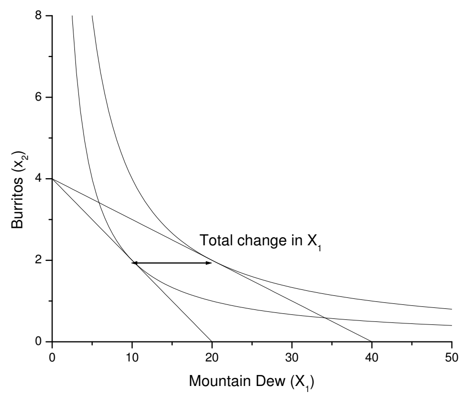
1. The utility function is equivalent to $U(x_1, x_2) = \ln x_1 + \ln x_2$. Using the "magic formula" for Cobb-Douglas preferences, find

$$x_1 = \frac{1}{2}\frac{m}{p_1} = \frac{1}{2}\frac{40}{1} = 20, x_2 = \frac{1}{2}\frac{m}{p_2} = \frac{1}{2}\frac{40}{10} = 2$$

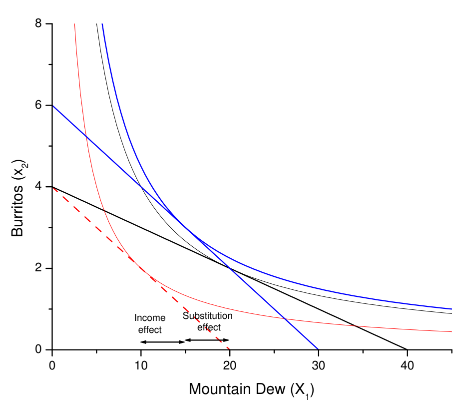
2. Only consumption of good 1 changes; the new consumption levels are

$$x_1 = \frac{1}{2}\frac{m}{p_1} = \frac{1}{2}\frac{40}{2} = 10, x_2 = 2$$

The total change in consumption of Mountain Dew equals -10:



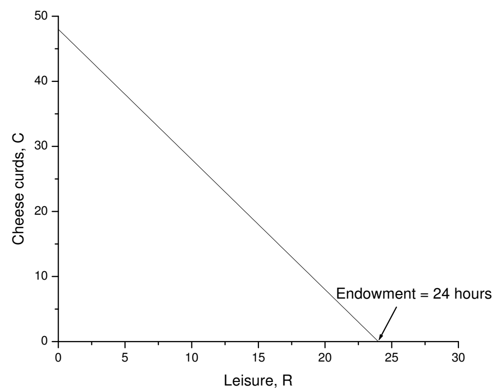
3. Auxiliary income $m' = p_1' x_1 + p_2 x2 = 2 \cdot 20 + 2 \cdot 10 = 60$, optimal consumption of Mountain Dew with auxiliary budget is $x_1 = \frac{1}{2}\frac{60}{2} = 15$, and the substitution effect $SE = 15 - 20 = -5$. The remaining change is due to the income effect, $IE = -10 - (-5) = -5$. On the graph:



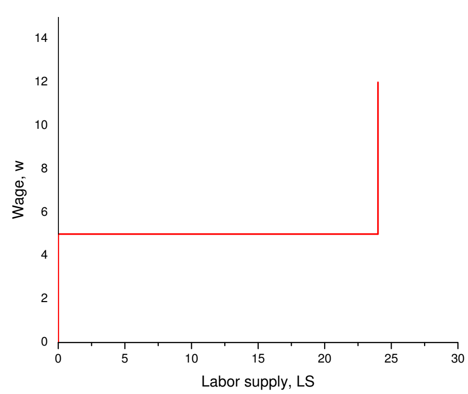
1.2 Perfect complements

1. $U(x_1,x_2) = \min\{2x_1, x_2\}$
2. Using the "magic formula" for perfect complements, find
$$x_1 = \frac{m}{p_1 + 2p_2} = \frac{40}{6+2\cdot 2} = 4, x_2 = 2x_1 = 8$$
This is an interior solution.
3. The new demand are $x_1 = 8, x_2 = 16$. The total change of 4 in consumption of left shoes comes from income effect (substitution effect for perfect complements is always zero), so $IE = 4$.

2 Labor-Leisure Choice



1.
2. Perfect substitutes (linear utility function)
3. The slope of budget line is 2, greater than the slope of indifference curves (1). Consumption is more valuable than leisure, so Jacob spends all his time working: $C = 48$, $R = 0$, $LS = 24$.
4. The switch from work to leisure happens when the budget line and indifference curves have the same slope:
$$\frac{w}{5} = 1 \Rightarrow w = 1$$



Problem 2 (15p) (Quasilinear Preferences)
a) Find marginal rate of substitution as a function of (x_1, x_2) (1 point).

$$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{4}{x_1}}{1} = \frac{4}{x_1}$$

b) Using two secrets of happiness find optimal consumption choices (4 points)

1)$20 = 4x_1 + 2x_2$

2)$\frac{4}{x_1} = \frac{4}{2} \implies x_1 = 2$

Plug x_1 = 2 into the first secret (budget constraint) to reveal: $x_1 = 2, x_2 = 6$

c) Suppose the price of a computer goes down to p_1= 2. Find optimal choice after the price change (1 point).

To find the new optimal choice repeat the above steps with $P_1 = 1$:
1)$20 = 2x_1 + 2x_2$

2)$\frac{4}{x_1} = \frac{2}{2} \implies x_1 = 4$

Plug x_1 = 4 into the first secret (budget constraint) to reveal: $x_1 = 4, x_2 = 6$

Decompose the change in x_1 into a substitution effect (2 points) and income effect (2 points).
To calculate the substitution and income effects we need to find an 'intermediate point'. This is the optimal consumption choice given the purchasing power necessary to buy the original optimal consumption bundle (2,6) given the new prices $P_1 = 2, P_2 = 2$:

M' = (2)2 + (2)6 $\implies$ M' = 116

To find the 'intermediate' optimal choice repeat the above steps with $P_1 = 2$ and M = 16:
1) $16 = 2x_1 + 2x_2$

2) $\frac{4}{x_1} = \frac{2}{2} \implies x_1 = 4$

Plug x_1 = 4 into the first secret (budget constraint) to reveal: $x_1 = 4, x_2 = 4$

The substitution effect is the change in consumption of x_1 due only to the price change (holding purchasing power constant). That means the substitution effect is the difference between the amount of x_1 originally consumed (2 units) and how much is consumed after the price change (4 units). The substitution effect is then 2 units of x_1, while the income effect is zero (when we increase purchasing power from M = 16 to M = 20 we do not change our level of x_1 consumption, only x_2.

d) Find optimal consumption for p_1= 4 , p_2= 2 and m = 4 (1 point). 1) $4 = 4x_1 + 2x_2$

2) $\frac{4}{x_1} = \frac{4}{2} \implies x_1 = 2$

Plug x_1 = 2 into the first secret (budget constraint) reveals that: $x_1 = 2, x_2 = -2$. We cannot consumer negative amounts of a consumption good, so this indicates we would prefer to give up 2 unit of x_2 in order to consume more x_1. This is not an option, however, as the lower bound on x_2 is zero.

Is your solution interior? (1 point).

Our solution is $x_1 = 1$, $x_2 = 0$, which is not interior (corner solution). Interior solutions are those at which consumption of each good is strictly greater than zero, which is not the case here.

Is marginal utility of a dollar equalized? (3 points)

No, we will very rarely see marginal utility per dollar of each consumption good equalized at a corner solution. In this case we have:

$$\frac{MU_{x_1}}{P_1} = \frac{\frac{4}{x_1}}{4} = \frac{4}{4} \neq \frac{1}{2} = \frac{MU_{x_2}}{P_2}$$