Discussion Section #5 Handout

February 21th

Present Value

The present value of any goods is the amount of present consumption that someone would give for it. PV of one unit of present consumption is 1. PV of one unit of consumption tomorrow is $\frac{1}{1+r}$.

Geometric Series

$$S_n = 0 + \frac{x}{1+r} + \frac{x}{(1+r)^2} + \cdots + \frac{x}{(1+r)^n} = \frac{x}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

If n is infinity, then $S_\infty = \frac{x}{r}$

Perpetuity: an asset which gives payment x forever. $PV^p = \frac{x}{r}$

Annuity: an asset which gives payment x for T periods. $PV^a = PV_0^p - PV_T^p = \frac{x}{r}\left[1 - \left(\frac{1}{1+r}\right)^T\right]$

Intertemporal Choice

Consumption today and consumption tomorrow are different goods, due to the interest rate r and discount rate δ.

$$\max \quad U(c_t, c_{t+1}) = u(c_t) + \frac{1}{1+\delta} u(c_{t+1})$$

$$s.t.\ c_t + \frac{c_{t+1}}{1+r} = m_t + \frac{m_{t+1}}{1+r} = PV(m_t, m_{t+1})$$

We can solve this problem by the ‘’two secrets of happiness”.

$$\begin{cases} \frac{MU_1}{P_1} = \frac{MU_2}{P_2} \\ P_1 x_1 + P_2 x_2 = m \end{cases} \rightarrow \begin{cases} u'(c_t) = \frac{u'(c_{t+1})(1+r)}{1+\delta} & (1) \\ c_t + \frac{c_{t+1}}{1+r} = m_t + \frac{m_{t+1}}{1+r} & (2) \end{cases}$$

Eq(1) is called **the Euler equation**.
Eq(2) is the **intertemporal budget constraint**.
Note that when $\delta = r,\ c_t = c_{t+1}$, we call this **consumption smoothing**.

Problem 1: Buying and Selling

Ramon Gonzales M. Panetelas is a specialist in Habanos cigars (famous Cuban cigars). Cuban cigars are sold either in packs of 10 cigars (x1), or in singles (x2). Ramon has no income. Instead he is initially endowed with w1 = 5 packs and w2 = 50 singles.

a. Draw Ramon’s budget set, given the price of a pack is equal to p1 = 5 and the price of a single cigar is p2 = 1. Mark the endowment point.
b. Illustrate geometrically his optimal demand for packs and single cigars, given his utility function U(x1, x2) = 10x1 + x2 (Give two numbers (x1, x2), and mark them on the graph, including the budget set and the indifference curve.)
c. What is your answer to the previous question when prices are p1 = 20 and p2 = 1? Give two numbers (x1, x2), and plot the graph.
d. (Harder) Give the formula for the demands x1, x2 as a function of p1, p2 and endowments w1, w2. Show the demand curve for x1 on the graph, assuming p2 = 1, w1 = 5, and w2 = 50.

Problem 2: Intertemporal Choice

Frank works as a consultant. His income is $2000 in period 1 but $8000 in period 2. Suppose that the interest rate is r = 100%.

a. Represent his budget line by a graph in the (C1, C2) plane.
b. Find analytically PV and CV of income and mark each in the plane.

Franks utility function is given by:

$$U(C_1, C_2) = \log(C_1) + \frac{1}{1+d}\log(C_2)$$

where the discount factor is d = 100%. Using the magic formula, find the optimal consumption plan and find how much Frank borrows or saves in the first period.

c. Is he smoothing his consumption?
d. Using the two secrets of happiness, show that his optimal consumption (C1, C2) decreases over time (that is, C1 > C2) when r < d.

Problem 3: Midterm 2010

a) Your sister just promised to send you pocket money of $500 each month starting next month and she will keep doing it forever. What's the present value of ''having a such sister'' if monthly interest rate is 5%.

b) Sam is a hockey player who earns $100 when young and $0 when old. Sam's intertemporal utility is

$$U(C_1, C_2) = \log(C_1) + \frac{1}{1+\delta}\log(C_2)$$

Assuming $\delta = r = 0$ and using magic formulas, find the optimal consumption plan and optimal saving strategy (Give three numbers C_1, C_2, S). Does Sam smooth his consumption?

c) You start the first job at the age of 21 and you work till 60, and then you retire. You live till 80. Your annual earnings between 21-60 are $100, 000 and the interest rate if *r* = 5%. You want to maintain a constant level of consumption. Write down an equation that determines *C*. (Write down one equation but you **do not need** to solve for *C*)