1 Buying and Selling

Midterm 2008 Spring

Ramon Gonzales M. Panetelas is a specialist in Habanos cigars (famous Cuban cigars). Cuban cigars are sold either in 10 cigar packs (x_1), or in singles (x_2). Ramon has no income. Instead he is initially endowed with $\omega_1 = 5$ packs and $\omega_2 = 50$ cigars.

(a) Draw his budget set, given the price of a pack is equal to $p_1 = \$5$ and a single cigar $p_2 = \$1$. Mark the endowment point.

ANSWER Since the valuation of his endowment is $5 \cdot 5 + 1 \cdot 50 = 75$, the budget line's equation is given by $5x_1 + x_2 = 75$.

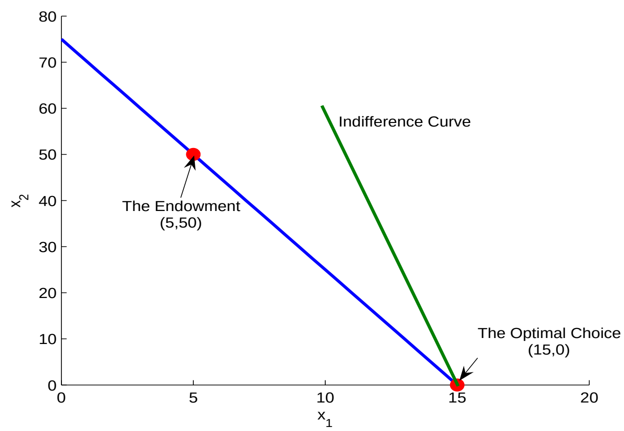

Figure 1: The Budget Line and the Optimal Choice

(b) Illustrate geometrically his optimal demand for packs and single cigars, given his utility function

$$U(x_1, x_2) = 10x_1 + x_2 \tag{1}$$

(Give two numbers (x_1, x_2), and mark them on the graph, including the budget set and the indifference curve.)

ANSWER Note that the two goods are perfect substitutes, and that good 1 presents the larger marginal utility from a dollar:

$$\frac{\text{MU}_1}{p_1} = \frac{10}{5} > \frac{1}{1} = \frac{\text{MU}_2}{p_2} \tag{2}$$

Therefore, he will consume good 1 only. $x_1 = 15$ but $x_2 = 0$. For choosing the bundle (15,0) he is remunerated

$$U = 10 \cdot 15 + 0 = 150.$$

So the equation of the indifference curve passing through the optimal bundle is $x_2 = 150 - 10x_1$. It is depicted by the green line in the figure.

(c) What is your answer to (b) when prices are $p_1 = 20$ and $p_2 = 1$. Give two numbers (x_1, x_2), and plot the graph.

ANSWER If p_1 increases to 20, the inequality in (2) would be reversed:

$$\frac{\text{MU}_1}{p_1} = \frac{10}{20} < \frac{1}{1} = \frac{\text{MU}_2}{p_2} \tag{3}$$

That is, good 2 now yields the larger marginal utility from a dollar. It is easy to see that he will consume good 2 only; however, income changes to m=20*5+1*50=150, so x2=150 .

(d) Harder: Give the formula for the demands x_1, x_2 as a function of p_1, p_2 and endowments ω_1 and ω_2. Show the demand curve for x_1 on the graph, assuming $p_2 = 1$, $\omega_1 = 5$, and $\omega_2 = 50$.

ANSWER

$$x_1 = \begin{cases} \frac{p_1\omega_1 + p_2\omega_2}{p_1} & \text{if } \frac{10}{p_1} > \frac{1}{p_2} \\ 0 & \text{if } \frac{10}{p_1} < \frac{1}{p_2}, \end{cases}$$

$$x_2 = \begin{cases} 0 & \text{if } \frac{10}{p_1} > \frac{1}{p_2} \\ \frac{p_1\omega_1 + p_2\omega_2}{p_2} & \text{if } \frac{10}{p_1} < \frac{1}{p_2}, \end{cases}$$

And in case $\dfrac{10}{p_1} = \dfrac{1}{p_2}$, any bundle on the associated budget line will be optimal. Observe that when $p_2 = 1$, $\omega_1 = 5$, and $\omega_2 = 50$ the above x_1 comes down to

$$x_1 = \begin{cases} \frac{5p_1 + 50}{p_1} & \text{if } p_1 < 10 \\ 0 & \text{if } p_1 > 10, \end{cases}$$

And if $p_1 = 10$ then the two goods produce the same marginal utility from a dollar. So any bundle on the budget line, $10x_1 + x_2 = 100$, would be optimal. That is, x_1 can take any value between 0 and 10. See figure 2.

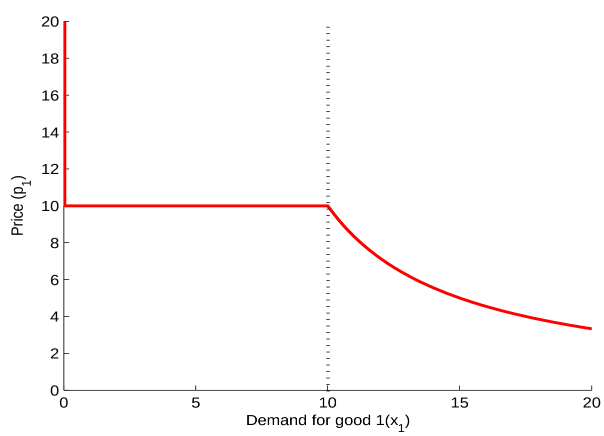

Figure 2: Demand Curve for good 1

2 Intertemporal Choice

Midterm 2009 Spring

Frank works as a consultant. His income is \$2000 in period 1 but \$8000 in period 2. Suppose that the interest rate is $r = 100\%$.

(a) Represent his budget line by a graph in the (C_1, C_2) plane.

ANSWER There are two ways to express the budget line: (1) in terms of present value and (2) in terms of future value.

(1) The one(and recommended) way is to translate both his expenditure and incomes into their present values. First, the present value of his income stream is written by $2000 + \frac{8000}{1+r} = 6000$. And we denote by C_1 and C_2 the amount of consumption in period 1 and 2, respectively. Then its present value is $C_1 + \frac{C_2}{1+r} = C_1 + \frac{1}{2}C_2$. Since the budget constraint(or line) simply equates these two things, it is given by

$$C_1 + \frac{1}{2}C_2 = 6000. \tag{4}$$

(2) Alternatively, we can express it in terms of future value. The future value of the income stream is $(1+r)2000 + 8000 = 12,000$. Hence

$$2C_1 + C_2 = 12,000 \tag{5}$$

It is easy to see that these two budget line equations do not make any mathematical difference. We can get the latter by multiplying the former by $1 + r = 2$. The only (economic) difference is the first budget line makes the price of C_1 equal to 1, while the second one makes the price of C_2 equal to 1. Refer to Figure 3.

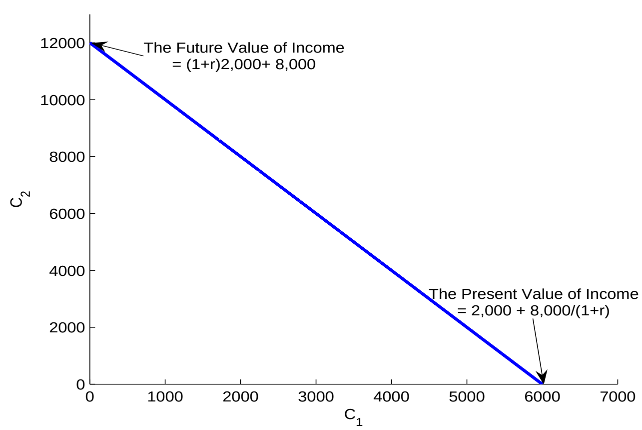

Figure 3: The Budget Line

(b) Find analytically PV and FV of income. And mark each in the plane.

ANSWER They are 6,000 and 12,000. Observe that its PV(FV) becomes the $x(y)$-intercept in Figure 2. So, for instance, the present value of income gives the maximum amount of first-period consumption(C_1) possible.

Frank's utility function is given by

$$U(C_1, C_2) = \log C_1 + \frac{1}{1+\delta} \log C_2 \tag{6}$$

where the discount factor is $\delta = 100\%$. Using the magic formula, find the optimal consumption plan and find how much he borrows or saves in the first period.

ANSWER Since his discount factor is given by $1 = 100\%$, we can rewrite his utility function by

$$U(C_1, C_2) = \log C_1 + \frac{1}{2} \log C_2 \tag{7}$$

It is of Cobb-Douglas with $a = 1$ and $b = \frac{1}{2}$. Using the magic formula and the budget constraint we got in the preceding part, we can write the optimal consumption plan C_1 and C_2 as

$$C_1 = \frac{a}{a+b}\frac{m}{p_1} = \frac{2}{3}\frac{6,000}{1} = 4,000$$
$$C_2 = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{3}\frac{6,000}{1/2} = 4,000.$$

Here, I plug in $m = 6,000$, that is, the present value of income, which means that I regard the price of C_1 as 1 and the price of C_2 as $\frac{1}{2}$. See part (a).

(c) Is he smoothing his consumption?

ANSWER Yes. It means that Frank is willing to have a stable path of con-sumption although his income fluctuates. The idea is related to Milton Friedman's *Permanent Income Hypothesis*, which basically argues that people's consumption behavior is determined not by current income but by their long-term income expectation.

(d) Using two secrets of happiness, show that his optimal consumption (C_1, C_2) decreases over time(that is, $C_1 > C_2$) when $r < \delta$.

On the contrary to the hypothesis, it is difficult to see the consumption smoothing behavior in real data. The reason is that it is based on many (unrealistic) assumptions. Here, we will see that it breaks down even when the interest rate is not equal to the discount factor, that is $r \neq \delta$. Allowing them to take any arbitrary value, we can write off two secrets of happiness as

$$(1)\quad C_1 + \frac{1}{1+r}C_2 = 2,000 + \frac{1}{1+r}8,000$$
$$(2)\quad \frac{1}{C_1} = \frac{1+r}{1+\delta}\frac{1}{C_2}$$

To wit briefly, the first condition states the budget constraint in terms of present value as we have seen above, and the second condition requires that the two marginal utilities from a dollar, generated by C_1 and C_2, agree. Solve the second condition for C_1 to obtain

$$C_1 = \frac{1+\delta}{1+r}C_2$$

From the assumption $r < \delta$, we can see that the fraction, $\frac{1+\delta}{1+r}$, is greater than 1. Therefore, $C_1 > C_2$.

Midterm 2010 Spring

(a) Your sister has just promised to send you pocket money of \$500 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5%.

ANSWER $\text{PV} = \frac{500}{0.05} = 10,000.$

(b) Sam is a hockey player who earns \$100 when young and \$0 when old. Sam's intertemporal utility is given by

$$U(C_1, C_2) = \log C_1 + \frac{1}{1+\delta}\log C_2. \qquad (8)$$

Assuming $\delta = r = 0$ and using magic formulas, find the optimal consumption plan and optimal saving strategy (Give three numbers C_1, C_2, S). Does Sam smooth his consumption?

ANSWER Note that the present value of income is 100. The magic formula gives

$$C_1 = \frac{1}{2}\frac{100}{1} = 50$$
$$C_2 = \frac{1}{2}\frac{100}{2} = 50.$$

It means that he saves \$50 in the first period.

(c) You start the first job at the age of 21 and you work till 60, and then you retire. You live till 80. Your annual earnings between 21-60 are $\$100,000$ and the interest rate if $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that determines C. (Write down one equation but you **do not need** to solve for C.

ANSWER The present value of earnings between 21 and 60 is

$$\frac{100,000}{0.05}\left[1 - \frac{1}{(1+0.05)^{40}}\right] \tag{9}$$

And the present value of (constant) consumption plan between 21 and 80 is

$$\frac{C}{0.05}\left[1 - \frac{1}{(1+0.05)^{60}}\right] \tag{10}$$

The consumption level C is determined by the equation that equates the two terms:

$$\frac{100,000}{0.05}\left[1 - \frac{1}{(1+0.05)^{40}}\right] = \frac{C}{0.05}\left[1 - \frac{1}{(1+0.05)^{60}}\right] \tag{11}$$