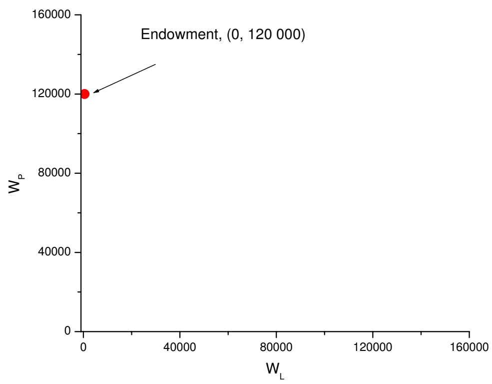
Uncertainty and Insurance; General Equilibrium

Econ 301: Week 7 Solutions

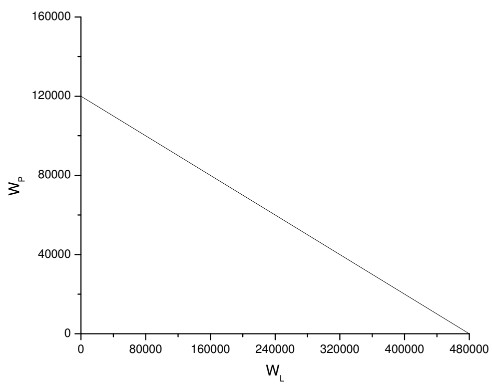
March 4, 2011

1 Choice of insurance

Example 1

1. 

2. If Karl buys x dollars of insurance, his wealth in case company makes profits is $W_P = 120,000 - 0.2x$, and his wealth if the company makes losses is $W_L = 0.8x$. Solving these two equations for x, get the budget constraint $W_p + 1/4W_L = 120,000$.



3. Karl's von Neuman Morgenstern utility function is $U(W_L, W_P) = \frac{1}{5}\log W_L + \frac{4}{5}\log W_P$. The first secret of happiness gives $\frac{W_P}{4W_L} = \frac{1}{4}$, or $W_P = W_L$, so Karl insures fully. Using the 2nd secret of

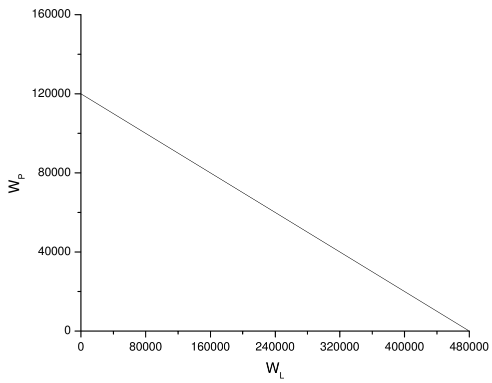
happiness, we have $W_p + 1/4W_P = 120,000 \Rightarrow W_P = W_L = 96,000$. The optimal level of insurance $x = W_L/0.8 = 96,000/0.8 = 120,000$

4. The budget constraint becomes $W_p + 2/3W_L = 120,000$; from the first secret of happiness get $\frac{W_P}{4W_L} = \frac{2}{3} \Rightarrow W_P = 8/3W_L$, Karl does not insure fully. From the second secret of happiness $8/3W_p + 2/3W_P = 120,000 \Rightarrow W_P = 96,000, W_L = 36,000$. The optimal level of insurance $x = W_L/0.6 = 36,000/0.6 = 60,000$

2 Risk aversion and certainty equivalence

Example 2

1. Bernoulli utility function is convex, so you are risk loving.

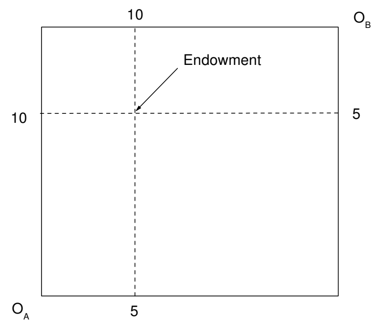


2. Expected value of the lottery is $E(L) = 19/20 \cdot 0 + 1/20 \cdot 3,000 = 150$, expected utility from the lottery $U(L) = 19/20 \cdot 0^2 + 1/20 \cdot 3,000^2 = 450,000$
3. Certainty equivalent is defined by $19/20 \cdot CE^2 + 1/20 \cdot CE^2 = 450,000 \Rightarrow CE \approx 670$
4. No, because the amount offered is lower than the certainty equivalent of the lottery
5. Your brother is risk averse. Expected utility from lottery for him is only $U(L) = 19/20 \cdot \sqrt{0} + 1/20 \cdot \sqrt{3,000} \approx 2.7$, and his certainty equivalent is $CE = 30/4$, so he will accept your offer.

3 General equilibrium and Edgeworth box

Example 3

1. The total resources are $w = (15, 15)$



2.

3. Pareto efficient allocations satisfy

$$MRS^A = MRS^B, C_M^A + C_M^B = 15, C_N^A + C_N^B = 15$$

Computing MRS, get

$$\frac{C_N^A}{2C_M^A} = \frac{C_N^B}{2C_M^B}$$

, so in Pareto efficient allocations $C_N^A = C_M^A$ and $C_N^B = C_M^B$ (Parteo efficient allocations are on the diagonal of Edgeworth box). Endowment allocation is not Pareto efficient.

4. Normalize $P_N = 1$, use restrictions on Pareto efficient allocations and the first secret of happiness for one of the consumers (say, Ann) to get

$$\frac{C_N^A}{2C_M^A} = \frac{P_M}{1} \Rightarrow P_M = \frac{1}{2}$$

Use the budget constraints to get equilibrium consumption:

$$\frac{1}{2}C_M^A + C_N^A = \frac{1}{2} \cdot 5 + 10 \Rightarrow C_M^A = C_N^A = \frac{25}{3}$$
$$\frac{1}{2}C_M^B + C_N^B = \frac{1}{2} \cdot 10 + 5 \Rightarrow C_M^B = C_N^B = \frac{20}{3}$$

Econ 301 Intermediate Microeconomics Spring 2013
Prof.Marek Weretka
Solution to Midterm 2 (Group A)

Problem 1. (a) expected utility function over lotteries

$$\begin{aligned} U(C_c, C_{nc}) &= \frac{1}{2} \times 10 \ln C_c + \frac{1}{2} \times 10 \ln C_{nc} \\ &= 5 \ln C_c + 5 \ln C_{nc} \qquad (3') \end{aligned}$$

Oscar is risk averse since $U(c) = 10 \ln C$ is concave. (1')

Let $U(C_c, C_{nc}) = \bar{u}$ Then $C_{nc} = \frac{e^{\bar{u}/5}}{C_c}$. So we can obtain the indifference curve as below. (1')

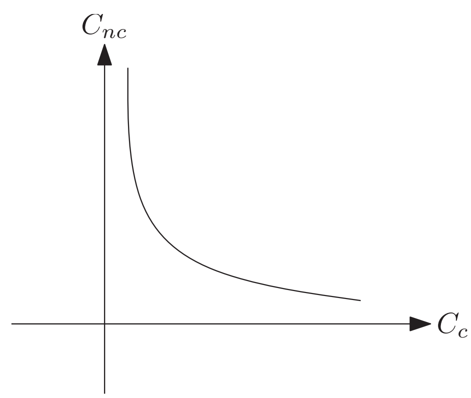


Figure 1: Problem 1(a)

(b) The C_c and C_{nc} can be represented as

$$\begin{aligned} C_c &= 4 - \gamma x + x = 4 + 0.5x \\ C_{nc} &= 8 - \gamma x = 8 - 0.5x \qquad (2') \end{aligned}$$

Then,

$$C_{nc} + \frac{\gamma}{1-\gamma} C_c = 8 + 4 \frac{\gamma}{1-\gamma} \Longrightarrow C_{nc} + C_c = 12 \qquad (3')$$

The budget line is shown as below: (2')

(c) By the utility function in part (a) and budget constraint in part (b), optimal choice of wealth levels are given by the short-cut formula:

$$C_c = \frac{5}{5+5} \times \frac{12}{1} = 6 \qquad (2')$$

$$C_{nc} = \frac{5}{5+5} \times \frac{12}{1} = 6 \qquad (2')$$

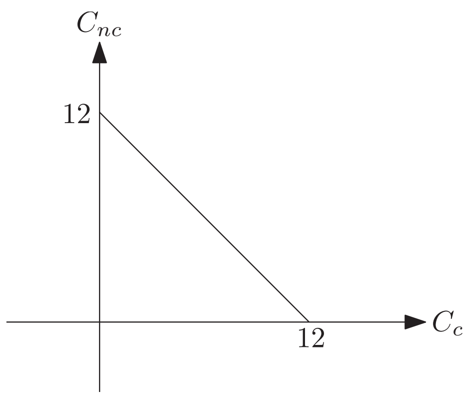


Figure 2: Problem 1(b)

Then $C_c = C_{nc}$ So Oscar is fully insured. (1')
By the formula in part(b), $x = 4$. (2')

(d) First calculate

$$MRS = \frac{5/C_{nc}}{5/C_c} = \frac{C_c}{C_{nc}} \qquad (2')$$

then by the 'secret of happiness',

$$MRS = \frac{p_{nc}}{p_c} \Longrightarrow \frac{C_c}{C_{nc}} = \frac{1-\gamma}{\gamma} \qquad (2')$$

If $\gamma > \pi_c = 0.5$, then $\frac{1-\gamma}{\gamma} < 1$. Therefor, $C_nc > C_c$. That is, Oscar is not fully insured. (2')

Problem 2. (a) Total endowment $\omega = \omega^E + \omega^B = (5, 30) + (20, 20) = (25, 50)$. (1')
The Edgeworth box and initial endowment are in the following figure.

(b) An allocation is Pareto Efficient if there is no other allocation which would make one of them strictly better off without hurting any of the others. (2')
$\Longrightarrow$) Suppose that $MRS^E \neq MRS^B$ when an allocation is Pareto Efficient. Then the two indifference curves passing through this point will intersect as in figure 4. It's obvious that the points in the shadowed area will make both of them not worse off and at least one of them strictly better off. That's a contradiction with the assumption. Then the necessity follows immediately. (3')
$\Longleftarrow$) Now assume the condition $MRS^E = MRS^B$ is satisfied at an allocation a. Then the two indifference curves passing through this point will be tangent with each other as in figure 5. Then check all

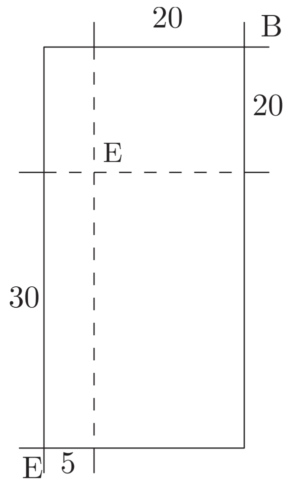


Figure 3: Problem 2(a)

the points in the areas A, B, C, D except point a in the Edgeworth box and all of them will make at least of Elisa and Ben strictly worse off. So allocation a is Pareto efficient by definition. (3')

(c)

$$MRS^E = MRS^B \Longrightarrow \frac{x_2^E}{x_1^E} = \frac{x_2^E}{x_1^E} \qquad (2')$$

Also

$$x_1^E + x_1^B = 25, x_2^E + x_2^B = 50 \qquad (2')$$

Substitute the last two equation to the first one.

$$x_2^E = 2x_1^E, x_2^B = 2x_1^E \qquad (2')$$

the contract curve is plotted in figure 6. (2')

(d) Normalize $p_2 = 1$. the budget constraints are:

$$\begin{aligned} 2x_1^E + x_2^E &= 5p_1 + 30 \\ 2x_1^B + x_2^B &= 20P_1 + 20 \qquad (1') \end{aligned}$$

Then

$$\begin{aligned} x_1^E &= \frac{2}{2+2}\frac{5p_1 + 30}{p_1} \\ x_1^B &= \frac{2}{2+2}\frac{20P_1 + 20}{p_1} \qquad (1') \end{aligned}$$

Thus

$$\frac{2}{2+2}\frac{5p_1 + 30}{p_1} + \frac{2}{2+2}\frac{20P_1 + 20}{p_1} = 25 \Longrightarrow p_1 = 2 \qquad (2')$$

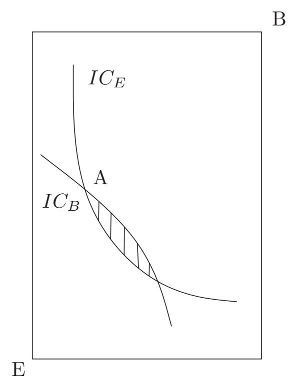


Figure 4: Problem 2(b)

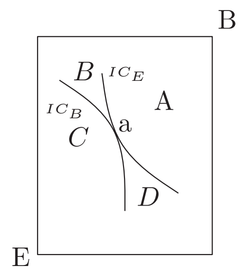


Figure 5: Problem 2(b)

Solve the equations and the result follows:

$$x_1^E = 10, x_2^E = 20 \qquad (2')$$

$$x_1^B = 15, x_2^B = 30 \qquad (2')$$

The competitive equilibrium is indicated in the Edgeworth box in Figure 7.

(e) Calculate and compare the MRS's.

$$MRS^E = \frac{x_2^E}{x_1^E} = \frac{20}{10} = 2 \qquad (1')$$

$$MRS^B = \frac{x_2^B}{x_1^B} = \frac{30}{15} = 2 \qquad (1')$$

So obviously $MRS^E = MRS^B$.

(f) any numbers satisfying $\frac{p_1}{p_2} = 2$, for example, $p_1 = 4, p_2 = 2$. (2')

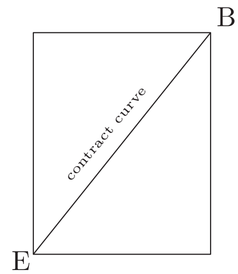


Figure 6: Problem 2(c)

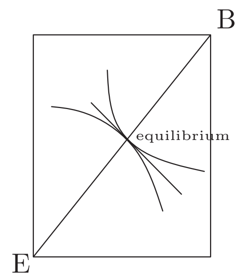


Figure 7: Problem 2(d)

Problem 3. (a) The production function $y = f(K,L) = \sqrt{K+L}$

$$f(rK, rL) = \sqrt{rK + rL} = \sqrt{r(K+L)} = \sqrt{r}\sqrt{K+L} = r^{\frac{1}{2}} f(K,L) < rf(K,L) \qquad (3')$$

So production function is decreasing returns to scale(DRS).

(b)

$$TRS = \frac{\partial y/\partial L}{\partial y/\partial K} = \frac{\frac{1}{2\sqrt{K+L}}}{\frac{1}{2\sqrt{K+L}}} = 1 \qquad (2')$$

Notice $\frac{\omega_L}{\omega_K} = 2$ so $TRS < \frac{\omega_L}{\omega_K}$. (2')
Hence $L = 0$ and $y = \sqrt{K} \Longrightarrow K = y^2$ (2')
Then $c(y) = 2 \times 0 + 1 \times y^2 = y^2$ (1')
The cost function is plotted by Figure 8. (1')

(c) $TC(y) = FC + VC(y) = 1 + y^2$ (1')

$$ATC(y) = \frac{TC(y)}{y} = \frac{1+y^2}{y} \qquad (1')$$

$$MC(y) = 2y \qquad (1')$$

To find minimum efficient scale(MES), let $MC(y) = ATC(y) \Longrightarrow 2y = \frac{1+y^2}{y} \Longrightarrow y^{MES} = 1$ (2')