Econ 301: Intermediate Microeconomics Spring 2014

Discussion Section #7 Handout March 14th

Example 1: Edgeworth box with Interest Rates

Andy and Bob have endowments of "income when young" (good 1) and "income when old" (good 2) given by $\omega^A = (600, 0)$ and $\omega^B = (0, 600)$. They both have the same utility function $U(x_1, x_2) = \ln(x_1) + b\ln(x_2)$, where the discount rate $b = 0.5$.

a) Draw an Edgeworth box and mark the endowment point. Is it Pareto efficient?

b) Find the equilibrium interest rate r (use $1 + r = p_1/p_2$) and the equilibrium consumption.

c) What are Andy's and Bob's saving/borrowing strategies given the equilibrium interest rate?

d) Characterize the equilibrium prices and consumption if the utility function was instead $U(x_1, x_2) = 3x_1 + 3x_2$

Example 2: Production Function

Consider the following production functions:

A. $f(K, L) = K^2L^2$
B. $f(K, L) = \sqrt{KL}$
C. $f(K, L) = K + 2L$

a) Sketch the isoquants for each production function.

b) Find MPL analytically for each of the functions. Plot MPL in a graph (L on x-axis, MPL on y-axis), assuming capital is fixed at K = 1.

c) For each function, show whether it exhibits increasing, constant, or decreasing returns to scale. Discuss.

Example 3: Profit Maximization (Short run)

Consider the following production functions:

$$f(K, L) = K^{1/4}L^{1/2}$$

In the short run, K cannot be adjusted and it is 81.

a) Derive short run production function and short run profit function.

b) Find MPL. What is the optimal condition for labor demand?

c) Find optimal labor demand as a function of wage w and output price p.

d) Draw labor demand function with respect to the real wage (w/P), when output price P=1.

[Past Exam Problems]

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (0, 50)$ and Bob's endowment is $\omega^B = (50, 0)$.

The utility function of both Andy and Bob is the same and given by

$$U(x_1, x_2) = 3 \ln x_1 + 3 \ln x_2$$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ...).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditiions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

g) Give some other prices that are consistent with competitive equilibrium (give two numbers).

f) Using MRS condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.