Handout #9 Solutions

1. Cost & Supply Functions

a) Without fixed cost, the production technology exhibits DRS.

With fixed cost, $c(y) = 0.5y^2 + 200$

$$ATC = 0.5y + \frac{200}{y}$$

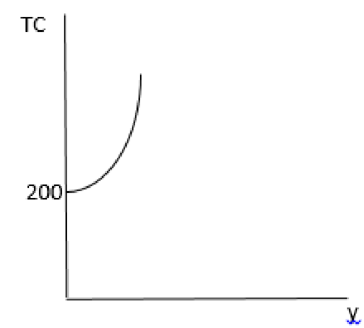
$$\frac{\partial ATC}{\partial y} = 0.5 - \frac{200}{y^2} = 0 \rightarrow y^{MES} = 20$$

So $\frac{\partial ATC}{\partial y}$ > 0 when y<20, IRS

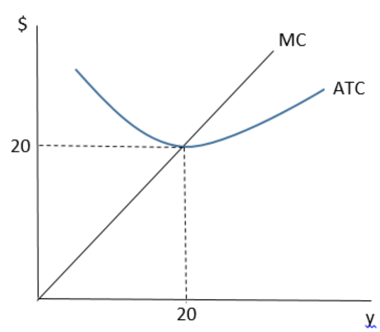
=0, when y=20, CRS

<0, when y>20, DRS

b) FC = 200, VC = $0.5y^2$ → TC = 200 + $0.5y^2$



c) AFC = 200/y, AVC = 0.5y, ATC = 00/y + 0.5y, MC = y → MC intersects ATC at the minimum point of the ATC



d) $MC(y^{MES}) = ATC(y^{MES})$ → y = 200/y + 0.5y → $0.5y^2 = 200$ → $y^2 = 400$ → $y^{MES} = 20$

You can also check this result by finding the minimum point of the ATC curve (setting the derivative of ATC equal to 0).

e) The profit of the firm is: $\pi(y) = TR(y) - TC(y) = py - 200 - 0.5y^2$

To maximize profit, set the derivative of profit w.r.t. y equal to 0:

$$\frac{\partial \pi}{\partial y} = \frac{\partial TR}{\partial y} - \frac{\partial TC}{\partial y} = 0$$

$$\frac{\partial TR}{\partial y} = \frac{\partial TC}{\partial y}$$
$$MR(y) = MC(y)$$

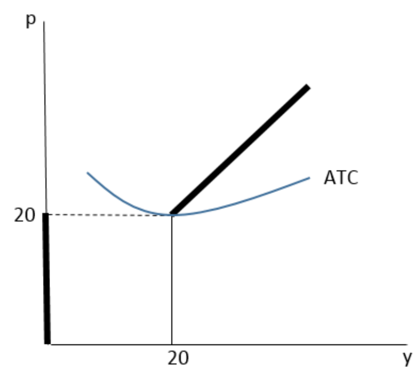
MR(y) = p
MC(y) = y

⇨ The profit-maximizing quantity y = p

So, for:
p = 10 → y = 10 → π(10) = 100 – 200 – 50 = -150
p = 20 → y = 20 → π(20) = 400 – 200 – 200 = 0
p = 30 → y = 30 → π(30) = 900 – 200 – 450 = 250

f) From above, the supply function is:

$$y(p) = \begin{cases} p, p \geq 20 \\ 0, p < 20 \end{cases}$$



2. Perfect Competition

a) MR = MC ➔ p = 5 + 2y → y = (p – 5)/2
The break-even price is p = ATC(y^{MES})
To find y^{MES}:
MC = ATC → 5 + 2y = 100/y + 5 + y → y^{MES} = 10
ATC(y^{MES}) = 100/10 + 5 + 10 = 25 ➔ the break-even price is p = 25
Then, the firm's supply function is:

$$y(p) = \begin{cases} \frac{p-5}{2}, p \geq 25 \\ 0, p < 25 \end{cases}$$

b) S(p) = N*y(p) ➔ $S(p) = \begin{cases} 5\,\frac{p-5}{2}, p \geq 25 \\ 0, p < 25 \end{cases}$

c) In equilibrium, S(p) = D(p)

5(p-5)/2 = 1000 – 10p → p* = 81 (eqm. price) → S(p*) = 190 (eqm. market quantity)→ y(p*) = S(p*)/N = 190/5 = 38 (eqm. firm quantity)
$\pi(y^*) = 81(38) - 100 - 5(38) - 38^2 = 1344$ (eqm. profit of each firm)

d) Free entry means that each firm makes zero profit in equilibrium, i.e., each firm produces at its minimal efficient scale. Then, $y^* = y^{MES} = 10$, p = 25.
Demand is D(25) = 1000 – 250 = 750 = S* and since each firm produces 10 units, the eqm. number of firms is N = S*/y* = 750/10 = 75 firms.
e) $c(y) = 5y + y^2 + F$, to find y_{MES}, we set MC=ATC, $5 + 2y = 5 + y + \frac{F}{y}$ → $y_{MES} = \sqrt{F}$
The break-even price is at $p^* = ATC_{MES} = 5 + 2\sqrt{F}$
The equilibrium is $S(p^*) = D(p^*)$
$N\frac{p-5^*}{2} = 1000 - 10p^*$, plug in $p^* = 5 + 2\sqrt{F}$, we get $N = \frac{950}{\sqrt{F}} - 20$

f)
when F=1444, N=5
F=361, N=30
F=100, N=75

Problem 3

$y = K^a L^b$, where a>1, b<1, and a+b>1.
Without calculation, we know ATC is decreasing with y when the production technology is increasing return to scale.