ECON 455, Discussion Section 11

TA: Shane Auerbach (sauerbach@wisc.edu) ; Date: 04/24/15

Office: SS 6470. OH: Wed 8:00-9:30am; Thu 4:15-5:45pm; or by appt.

1. (Transference) Suppose people lump all vehicles (in particular, airplanes (a), boats (b), and cars (c)) into one product category. Further suppose there are two types of consumers, which we'll refer to as the "unwashed masses (M)" and the "one-percenters (O)" respectively. While both groups lump together the three types of vehicles, the three vehicles have different salience levels to each group as follows:

$$p^M = (p_a^M, p_b^M, p_c^M) = \left(0, \frac{1}{4}, \frac{3}{4}\right), \quad p^O = (p_a^O, p_b^O, p_c^O) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$$

   Let q_a, q_b, and q_c denote the quality of an airplane, boat and car respectively, and consider that they are a function of wing length (x), pontoon circumference (y) and radio antenna length (z) respectively in the following way:

$$q_a = 3x, \quad q_b = 2y, \quad q_c = z$$

   Assume that each vehicle will sell for its perceived quality, and that the costs of building wings, pontoons and radio antennas are:

$$c(x) = x^2, \quad c(y) = 2y^2, \quad c(z) = 3z^2$$

   (a) For just this part, assume that agents actually don't group the three vehicles. Find the optimal x, y, z (to maximize profits) for each of a, b, and c.

   (b) Going forward, assume the agents do group the three vehicles into one category. Explain why we might want to put pontoons and/or wings onto cars?

   (c) You are asked to design the optimal car (in that it maximizes profits) to sell to the masses. What is it?

   (d) Find the optimal (in that it maximizes profits) boat to sell to the masses (hint: you do not need to do any math).

   (e) Find the optimal airplane/boat/car to sell to the one-percenters (O).

   (f) Finally, suppose half of the population are type O (as illogical as that may be) and half are type M and you can only design one boat that you must sell to both types. Find the optimal (i.e. profit-maximizing) boat.

   (g) Suppose that agents can actually tell the difference between boats and airplanes, but aren't sure whether a car is really closer to an airplane or a boat. A recent study came out that said that air travel was significantly safer and a lot more fun than boating, even more so than thought previously. If you were building cars, describe (qualitatively) how this might affect your choice of x, y, z? Use a framing argument.

2. Martha is a quasi-hyperbolic discounter with $\beta = 1/5$ and $\delta = 1$. Her true ability, θ may be either $\theta = 1$ or $\theta = 3$ and she can put effort $e \in [0, 1]$ into studying for a course. If she does well in the course, it will generate a future payoff of 5. If she does poorly, it will generate a future payoff of 0. The probability she does well is $\frac{\theta e}{4}$. The cost of expending effort $e \in [0, 1]$ is ce, where she discovers c during the course but she believes beforehand that c is drawn from a uniform distribution on $[0, 1]$.

   (a) Let $\pi_a(\theta, e, c)$ denote her expected payoff at the time of enrollment. Find it:

   (b) Argue that, given our assumptions, $\pi_a(\theta, e, c)$ is increasing in e so $e^* = 1$ is optimal.

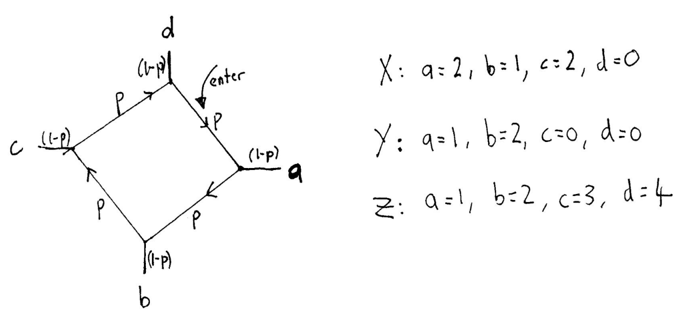
   (c) Let $\pi_b(\theta, e, c)$ denote her expected payoff during the course. Find it:

   (d) Let $E[\theta] = \hat{\theta}$. Obviously $\hat{\theta} \in [1, 3]$, and where it lies exactly depends on how Martha believes it's distributed. Find the optimal e^* for every possible $\hat{\theta}$, and c.

   (e) Suppose $E[\theta] = \hat{\theta} = 5/2$. What probability does enrollment-Martha put on course-Martha exerting effort $e = 1$?

   (f) $E[\theta] = \hat{\theta} = 5/2$ implies that $P[\theta = 3] = \frac{3}{4}$ and $P[\theta = 1] = \frac{1}{4}$]. Suppose Martha could take a costless test at enrolment time which would reveal θ to her during the course (before she chooses e). With probability $3/4$ it will reveal $\theta = 3$ and with probability $1/4$ it will reveal $\theta = 1$. If Martha signs up for the test, what probability does enrollment-Martha put on course-Martha exerting effort $e = 1$? Should Martha take the test?

3. (Forgetful driver challenge question) You're driving clockwise around a roundabout (traffic circle). You're a bit forgetful in that you can't tell the four exits on the roundabout from eachother. You know where you got on, but you can't see it each time you pass it, so you can't use that as a point of reference, and you don't remember how many exits you've already passed. Since you can't tell the exits apart, all you can do is choose p where, at each exit, p is the probability you stay on the roundabout and $(1 - p)$ is the probability you get off. All of this is shown in the picture below (albeit with a rather square roundabout). You're planning (in an *ex ante* sense) what p would be best for this situation.



   Note that X, Y, Z describe three different iterations of this situation, each with different payoffs at exits a, b, c, d.

   (a) Use intuition to find the optimal p for roundabout X.

   (b) Use intuition to guess an approximately optimal p for roundabout Y.

   (c) Use intuition to argue that there is no optimal p for roundabout Z. But also argue that we know roughly what that optimal p should be.

   (d) Set up the equation that you would maximize to solve for the optimal p.