ECON 455, Discussion Section 1

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Office: SS 6470. OH: Wed 8:00-9:30am; Thu 4:15-5:45pm; or by appt.

1. (Blind dates experiment) Pick from each of the following blind date scenarios:

   (a) **Blind Date Choice A**

   Your friend is offering to set you up with either Adam/Anna or Bill/Brenda:

Adam/Anna:	very intelligent, plain-looking, well-off
Bill/Brenda:	intelligent, very good-looking, poor

   Which one would you pick? Keep track of the number 1 if Adam/Anna, 2 if Bill/Brenda.

   (b) **Blind Date Choice B**

   Your first date didn't work out, so your friend offers to set you up again:

Eric/Elizabeth:	fairly intelligent, good-looking, rich
Christian/Christina:	intelligent, very good-looking, poor

   Next to your first digit, write down 1 if you pick Christian/Christina, 2 if Eric/Elizabeth (so you now have a 2-digit #)

   (c) **Blind Date Choice C**

   Your second date also didn't work out, so one more try:

Ricardo/Rebecca:	fairly intelligent, good-looking, rich
Nathan/Naomi:	very intelligent, plain-looking, well-off

   Now add a third digit to the number you have so far: 1 if Ricardo/Rebecca, 2 if Nathan/Naomi.

   Solution:

   I didn't put the question on the handout because I didn't want to hint at what we were aiming for. But the question is: **Would any set of choices, represented by a 3-digit number, violate rationality?**

   Indeed, they would. But first, here are the experimental and class results:

Theme		301	302	303	304	305	Class Total (%)	Original Experiment
	111	0	1	3	0	0	4 (5%)	18%
Int.	112	2	6	3	2	2	15 (19%)	40%
Mon.	121	2	1	3	3	3	12 (15%)	23%
	122	1	1	4	1	2	9 (11%)	2%
Att.	211	4	0	7	5	4	20 (25%)	0%
	212	3	1	2	1	2	9 (11%)	11%
	221	0	2	4	1	2	9 (11%)	6%
	222	0	1	0	0	1	2 (3%)	0%

   Let's think about what 111 means. It means, you prefer:

   (very intelligent, plain-looking, well-off) over (intelligent, very good-looking, poor),

   (intelligent, very good-looking, poor) over (fairly ingelligent, good-looking, rich)

   (fairly intelligent, good-looking, rich) over (very intelligent, plain-looking, well-off)

If we simply label the three descriptions by their first letters v, i, and f, then we have $v \succ i \succ f \succ v$, so transitivity is violated. In the second choice, we had you assign 1 to the second option rather than the first just to mask the transitivity a little. Similarly, 222 violates transitivity because $v \succ f \succ i \succ v$.

Transitivity: If $x \succ y$ and $y \succ z$, then $x \succ z$.

One way to get to 111 as your choice is to pick whichever person is best in 2/3 of the attributes in each case.

111 and 222 also violate the following assumption included in rationality (*technical note*: only if we assume you are not indifferent between any two possible dates, i.e. a strict preference ordering):

Independence of irrelevant alternatives (IIA): If $y \notin C(\{x, y\})$, then $y \notin C(\{x, y, z\})$.

In words, if you prefer x from $\{x, y\}$, then you should not prefer y from $\{x, y, z\}$ – z is the irrelevant alternative. An alternative formulation of IIA, one that we will use here, is if $C(\{x, y, z\}) = \{x\}$, then $C(\{x, y\}) = \{x\}$.

Here's how 111 violates IIA:

- $111 \equiv \{C(\{v, i\}) = \{v\}, C(\{i, f\}) = \{i\}, C(\{f, v\}) = \{f\}\}$
- One of the following must be true (given technical assumption above):
  - $C(\{v, i, f\}) = \{v\} \;\Rightarrow\; C(\{v, f\}) = \{v\}$, not true here
  - $C(\{v, i, f\}) = \{i\} \;\Rightarrow\; C(\{v, i\}) = \{i\}$, not true here
  - $C(\{v, i, f\}) = \{f\} \;\Rightarrow\; C(\{i, f\}) = \{f\}$, not true here
- No matter what he would pick if he was presented with all three options at once, IIA implies a choice from 2-element sets that he violates

Finally, I should mention the other assumption in rationality:

Completeness: All options can be ranked in a total order of preference.

A total order means that we can compare any two objects with eachother. You'll sometimes see it described as a complete partial order, also. It does not mean that a person must have a strict preference between any two options (that would be a strict order, as we assumed above) – you may be indifferent between two things and still be rational, so long as you can compare them.

The three assumptions boxed above together define rationality.

2. (Speeding) There are n cars on the road, including you, each driving at w_i ($i = 1 \ldots n$) miles per hour. Suppose you are agent $i = 1$. The average speed of traffic is $\bar{w} = \frac{1}{n} \sum_i w_i$, and no car can go faster than 100 mph. The probability that a driver gets in a crash is $p = \frac{w_i \cdot \bar{w}}{10{,}000}$. Suppose n is large enough that $\frac{\partial \bar{w}}{\partial w_i} \approx 0$, i.e. there are enough cars that your speed doesn't materially affect the average. If a driver crashes, he destroys his car, costing him a dollar

amount (the deductible on the insurance) of x. However, drivers like going fast – a driver's utility (in dollars) is $u_i(w_i) = 1000 \log w_i$. Assume drivers are risk neutral.

(a) Find your optimal speed as a function of the average speed and the deductible. Why is it increasing/decreasing in those variables?

Solution: Your problem is the following:

$$\max 1000 \log w_1 - \frac{w_i \cdot \bar{w}}{10000} \cdot x$$

Taking the FOC gives:

$$\frac{1000}{w_1^*} - \frac{\bar{w}x}{10000} = 0$$

And solving:

$$w_1^* \bar{w} x = 10000000$$
$$w_1^* = \frac{10000000}{\bar{w}x}$$

(b) Suppose a symmetric equilibrium, i.e. $w_i = w_j = \bar{w}$. If the policy maker wants all drivers to drive 65mph, what is the insurance deductible x that she would impose.

Solution: We simply solve

$$65 = \frac{10000000}{65x}$$
$$x = 2367$$

3. (Review of constrained optimization) Do the following problems as instructed:

$$\mathbf{A} : \max_{c_1, c_2} 2 \log c_1 + \log c_2 \ \ s.t. \ \ c_2 = 1 - c_1 \qquad \mathbf{B} : \max_{y} 32 \log y - y^2 \ \ s.t. \ \ y \leq b$$

(a) Solve **A**.

Solution: Sub the constraint into the problem, so you have:

$$\max_{c_1} 2 \log c_1 + \log(1 - c_1)$$

The first order condition is:

$$\frac{2}{c_1^*} - \frac{1}{1 - c_1^*} = 0$$
$$c_1^* = 2 - 2c_1^*$$
$$c_1^* = 2/3$$

You could check a second order condition, but it's pretty clear the objective is concave.

(b) Solve **B** with $b = 6$.

Solution: When you have inequality constraints, the constraint is either going to bind (i.e. $y = b$ at the optimum) or it will not bind (i.e. $y < b$ at the optimum). If the latter is case, then we can ignore the constraint entirely and solve the problem unconstrained. Of course, you don't know *ex ante* whether or not the constraint binds. The simplest way to solve these problems is to just *hope* the constraint doesn't bind. If you then solve the unconstrained problem and the solution does not violate the constraint, then that solution is also the solution to the constrained optimization problem.

Solving the unconstrained problem, the FOC is:

$$\begin{aligned} \frac{32}{y} - 2y &= 0 \\ 2y^2 &= 32 \\ y^* &= 4 < 6 = b \end{aligned}$$

Since the constraint is met, $y^* = 4$ solves the constrained problem also.

(c) Solve **B** with $b = 2$.

Solution: Here, we already know the solution to the unconstrained problem, $y^* = 4$, does not meet the constraint. Since the objective is increasing on $[0, 2]$ – you can graph it to see if you need convincing – you want to set y as high as possible, i.e. $y^* = b = 2$, meaning that the constraint *does* bind in this case.

4. (Allais Paradox) Choose between the following two lotteries, L1 and L2:

L1 pays one million with probability (w.p.) 1 $\qquad$ **L2** pays $\begin{cases} \text{One million} & \text{w.p. } 0.89 \\ 0 & \text{w.p. } 0.01 \\ \text{Five million} & \text{w.p. } 0.10 \end{cases}$

Now choose between the following two lotteries, L3 and L4:

L3 pays $\begin{cases} \text{One million} & \text{w.p. } 0.11 \\ 0 & \text{w.p. } 0.89 \end{cases}$ $\qquad$ **L4** pays $\begin{cases} \text{Five million} & \text{w.p. } 0.10 \\ 0 & \text{w.p. } 0.90 \end{cases}$

Argue that choosing $L1$ in the first choice and $L4$ in the second is inconsistent with expected utility theory.

Solution: The first choice, **L1** over **L2**, given expected utility theory implies the following:

$$U(\text{One million}) > 0.89 \cdot U(\text{One million}) + 0.01 \cdot U(0) + 0.10 \cdot U(5 \text{ million}) \qquad (1)$$

The second choice, **L4** over **L3**, implies

$$0.11 \cdot U(\text{One million}) + 0.89 \cdot U(0) < 0.10 \cdot U(\text{Five million}) + 0.90 \cdot U(0) \qquad (2)$$

However, if we simply add $0.89 \cdot U(0) - 0.9 \cdot U(\text{One million})$ to both sides of (1), it directly contradicts (2).

The choice that we showed above is inconsistent with Expected Utility Theory is pretty commonly observed when this experiment is run.