ECON 455, Discussion Section 2

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Office: SS 6470. OH: Wed 8:00-9:30am; Thu 4:15-5:45pm; or by appt.

1. (Colonoscopies) (*Modified from Angner 9.4*) Most colon cancers develop from polyps. Because early screening can detect polyps before they become cancerous, colonoscopies are a good idea for adults. The problem with colonoscopies, however, is that they're unpleasant. Consider a model with three time periods, $t = 0$ (youth), $t = 1$ (adult), and $t = 2$ (elderly). Anne and Bob may choose between the following two options in period $t = 1$:

   - **Colonoscopy (C)** Have one in $t = 1$ ($u_1^C = 0$) and be healthy in $t = 2$ ($u_2^C = 18$)
   - **No colonoscopy (N)** Avoid it in $t = 1$ ($u_1^N = 6$) and be unhealthy in $t = 2$ ($u_2^N = 0$)

   Anne discounts the future exponentially with $\delta = 2/3$. Bob discounts the future quasi-hyperbolically with $\beta = 1/6$ and $\delta = 1$.

   (a) Is it irrational (generally speaking, not just for Anne and Bob) to avoid the colonoscopy?

   Solution: Of course it's not irrational... If δ is low, avoiding the colonoscopy would maximize utility. For instance, maybe you know you'll die before you ever get to $t = 2$ anyway (which would give you a δ of zero) – obviously in that case it's perfectly rational (and reasonable) to avoid.

   (b) At $t = 0$, what are Anne's present values for C and N?

   Solution:

$$PV_0^A(C) = \delta u_1^C + \delta^2 u_2^C = \frac{2}{3}0 + \left(\frac{2}{3}\right)^2 18 = \frac{4}{9} \cdot 18 = 8$$

$$PV_0^A(N) = \delta u_1^N + \delta^2 u_2^N = \frac{2}{3}6 + \left(\frac{2}{3}\right)^2 0 = \frac{2}{3} \cdot 6 = 4$$

   (c) At $t = 1$, what are Anne's present values for C and N?

   Solution:

$$PV_1^A(C) = u_1^C + \delta u_2^C = 0 + \frac{2}{3}18 = 12$$

$$PV_1^A(N) = u_1^N + \delta u_2^N = 6 + \frac{2}{3}0 = 6$$

   (d) At $t = 0$, what are Bob's present values for C and N?

   Solution:

$$PV_0^B(C) = \beta\delta u_1^C + \beta\delta^2 u_2^C = \frac{1}{6}0 + \frac{1}{6}18 = 3$$

$$PV_0^B(N) = \beta\delta u_1^N + \beta\delta^2 u_2^N = \frac{1}{6}6 + \frac{1}{6}0 = 1$$

   (e) At $t = 1$, what are Bob's present values for C and N?

   Solution:

$$PV_1^B(C) = u_1^C + \beta u_2^C = 0 + \frac{1}{6}18 = 3$$

$$PV_1^B(N) = u_1^N + \beta u_2^N = 6 + \frac{1}{6}0 = 6$$

(f) Shane's Colonoscopy Kidnapping Company (SCKC) offers a service in which clients in $t = 0$ pay Shane to kidnap them in $t = 1$, drug them (safely, of course), and forcibly take them to their doctors for a colonoscopy. In period $t = 0$, assuming both agents are sophisticates, what is the maximum Anne would be willing to pay for that service? What about Bob? (Think dollars = utils)

Solution: Anne would not be willing to pay anything for the service since she knows she will end up getting the colonoscopy in $t = 1$ whether or not she is forced into it. Bob, however, sees that he will not get the colonoscopy in $t = 2$. Since $PV_0^B(C) = PV_0^B(N) + 2$, the maximum he would be willing to pay is 2.

2. (Calculating β and δ Q1)(*Modified from Angner 9.5*) Suppose we know that Martha is a quasi-hyperbolic discounter and that she is indifferent between one util today and three utils tomorrow. We also know that Clive (also a quasi-hyperbolic discounter) is indifferent between 1 util tomorrow and 3 utils the day after tomorrow.

(a) If Martha's $\beta = 1/2$, what is Martha's δ?

Solution:

$$1 = \frac{1}{2}\delta \cdot 3 \;\;\Rightarrow\;\; \delta = \frac{2}{3}$$

(b) If Martha's $\delta = 4/9$, what is Martha's β?

Solution:

$$1 = \frac{4}{9}\beta \cdot 3 \;\;\Rightarrow\;\; \beta = \frac{3}{4}$$

(c) Explain why we cannot determine Clive's β even if we're given his δ?

Solution: We get the following equation:

$$\beta\delta \cdot 1 = \beta\delta^2 \cdot 3$$

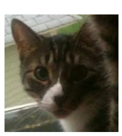
We cannot work out β from this equation because it just cancels on both sides.

(d) What is Clive's δ?

Solution: We can use that equation to solve for δ:

$$\delta \cdot 1 = \delta^2 \cdot 3 \;\;\Rightarrow\;\; \delta = \frac{1}{3}$$

3. (Calculating β and δ Q2)(*Modified from Angner 9.7*) Sobaka (my cat) is a quasi-hyperbolic discounter with $\beta, \delta \in (0, 1)$. Given that today he's indifferent between the following three options (A, B and C), calculate his β and δ.



- A: 2 fish today
- B: 5 fish tomorrow
- C: 10 fish the day after tomorrow

Solution: Two unknowns, two equations. Actually we could get three equations, $A \sim B$, $B \sim C$ and $A \sim C$. But the third would be implied by the first two (i.e. not linearly independent), so wouldn't help. So we just take any two of the three:

$$A \sim B \;\Leftrightarrow\; 2 = 5\beta\delta \;\Leftrightarrow\; \beta = \frac{2}{5\delta}$$

$$B \sim C \;\Leftrightarrow\; 5\beta\delta = 10\beta\delta^2 \;\Leftrightarrow\; \delta = \frac{1}{2}$$

The second equation gave us δ directly. Plugging that into the first gives us $\beta = \frac{4}{5}$.

4. (The Sophisticate's Curse)(*Modified from Angner 9.8/9*) Your movie theater's schedule over the next four days is as follows – your utility from attending each movie is included in parentheses.

- Day 0 (Today): **F**ifty Shades of Grey ($u_0 = 3$)
- Day 1: **T**he Interview ($u_1 = 5$)
- Day 2: **W**hiplash ($u_2 = 8$)
- Day 3: **B**oyhood ($u_3 = 13$)

For all questions below, suppose $\delta = 1$ and $\beta = 1/2$, and QH stands for quasi-hyperbolic.

(a) If you are an exponential discounter and can go to three movies, which do you skip?

Solution: You could be pedantic and see what the exponential discounter would do in each time period. However, the whole point of exponential discounting is that it is time consistent, so we can just check the preferences today and those preferences will be consistent over time. Also note that $\delta = 1$, so the utilities listed above are the discounted present values, so you'll obviously skip the one that gives you the lowest utility, meaning you'll skip today.

(b) If you are a naive QH discounter and can go to three movies, which do you skip?

Solution: Evaluating the options today, you will get

$$PV_0(F) = 3 \;\; PV_0(T) = 2.5 \;\; PV_0(W) = 4 \;\; PV_0(B) = 6.5$$

Since going to T has the lowest PV, you will plan on skipping that and go to F today. When tomorrow (Day 1) comes, however, you'll evaluate the options as follows:

$$PV_1(T) = 5 \;\; PV_1(W) = 4 \;\; PV_1(B) = 6.5$$

Now W looks the least attractive, so you'll again decide to go to the movie to watch T, deviating from your plan on the previous day. On Day 2, you look at the options:

$$PV_2(W) = 8 \;\; PV_2(B) = 6.5$$

And now it looks like W is the better option. So you end up going to the first three movies and skipping the last movie, which happened to be the best. And you kept changing your plan, you silly naif!

(c) If you are a sophisticated QH discounter and can go to three movies, which do you skip?

Solution: Remember we solve these backwards. Obviously, on Day 3, you will go to B if and only if you have not already gone to three movies.

As for day 2. If you have already skipped a movie, obviously you're going to all subsequent movies. If you haven't already skipped a movie, however, you will face the same decision as the naif in (a):

$$PV_2(W) = 8 \;\; PV_2(B) = 6.5$$

And therefore you would choose to go to W instead of waiting for B.

In Day 1, you know that if you haven't already skipped a movie and you don't skip one today, then you will end up going to W. If you do skip today, then you will go to all subsequent movies. Therefore, by skipping today, you would allow yourself to see B on day 3 instead. Since $5 < \frac{1}{2}13$, you would skip in day 1 if you had not already skipped.

In Day 0, you know that if you do not skip today, you will skip tomorrow. Since F today is worth more than T tomorrow ($3 > \frac{1}{2}5$), you go today.

In conclusion, the sophisticated QH discounted skips T.

(d) If you are an exponential discounter and can go to <u>one movie</u>, which do you go to?

Solution: Same logic as (a). You go to B.

(e) If you are a naive QH discounter and can go to <u>one movie</u>, which do you go to?

Solution: On day zero, evaluating the options gives:

$$PV_0(F) = 3 \;\; PV_0(T) = 2.5 \;\; PV_0(W) = 4 \;\; PV_0(B) = 6.5$$

So you skip today (because W and B are better). On Day 1:

$$PV_1(T) = 5 \;\; PV_1(W) = 4 \;\; PV_1(B) = 6.5$$

Again, you skip T, as B is the better option. Then on Day 2:

$$PV_2(W) = 8 \;\; PV_2(B) = 6.5$$

So you end up going to just W.

(f) If you are a sophisticated QH discounter and can go to <u>one movie</u>, which do you go to?

Solution: Again, solve backwards. If you get a choice in Day 3, it's because you haven't already gone, so obviously you go.

In Day 2, if you get a choice, you haven't already gone. Since

$$PV_2(W) = 8 > PV_2(B) = 6.5$$

, you would go to W.

In Day 1, you know that if you skip T, you'll end up going to W. Since

$$PV_1(T) = 5 > PV_1(W) = 4$$

, you would go to T.

In Day 0, you know that if you skip F, you'll end up going to T. Since

$$PV_0(F) = 3 > PV_0(T) = 2.5$$

, you would go to F. Therefore, the sophisticate goes today.

(g) If you are the sophisticate in (c) and could, today, whack yourself in the head to make yourself a naif, would you have incentive to do so?

Solution: No. Check PV_0 of the naif's consumption plan versus the sophisticates. You'll see that the sophisticate in $t = 0$ prefers his consumption plan to that of the naif.

(h) If you are the sophisticate in (f) and could, today, whack yourself in the head to make yourself a naif, would you have incentive to do so?

Solution: Yes. The naif ends up going to W whereas the sophisticate goes to F. Since $PV_0(W) = 4 > 3 = PV_0(F)$, the sophisticated you would benefit from being naif. Apparently this is called preproperation, kinda the opposite of procrastination. A sophisticated type succumbs to temptation early because he knows that he will inevitably succumb in the next period even if he is able to avoid it today.