ECON 455, Discussion Section 3

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Office: SS 6470. OH: Wed 8:00-9:30am; Thu 4:15-5:45pm; or by appt.

1. (Cutoffs - job search) You've just graduated. Congrats! You're now on the job market. Each period you receive one wage offer, w_t, where each wage offer is an independent and identically distributed random variable drawn from a uniform distribution over $[0,1]$. If you accept the offer, you get w_t in that period and that same wage for the rest of time – assume you live infinitely. If you don't accept, you get zero and go into the next period and get another wage offer. The process repeats until you eventually accept. You are an exponential discounter, with discount rate $\delta \in (0,1)$. Let U denote the expected value of entering a period unemployed and V the value of being employed. *Note: This isn't a behavioral question, but it's a good exercise in cutoff strategies and conditional expectations.*

   (a) What is V as a function of the accepted wage, w?

   Solution:

$$V = \frac{w}{1-\delta}$$

   (b) Suppose your strategy is to accept any wage offer greater than w^*. Write U as a function of w^* and δ.

   Solution:

$$\begin{aligned}
U &= \Pr(\text{Accept } w_t)(\mathbf{E}[V|\text{Accept}]) + \Pr(\text{Reject } w_t)\delta U \\
U &= (1-w^*)\left(\left(\frac{w^*+1}{2}\right)\frac{1}{1-\delta}\right) + w^*\delta U \\
U(1-w^*\delta) &= (1-w^*)\left(\left(\frac{w^*+1}{2}\right)\frac{1}{1-\delta}\right) \\
U &= \left((1-w^*)\left(\left(\frac{w^*+1}{2}\right)\frac{1}{1-\delta}\right)\right)/(1-w^*\delta)
\end{aligned}$$

   (c) Suppose you are unemployed and want to choose your cut-off to maximize your expected utility. To do this, you should obviously set $\frac{\partial U}{\partial w^*} = 0$. It's not an easy operation, so let's outsource by plugging into www.wolframalpha.com the following:

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d/dw (((1-w)((\frac{w+1}{2})\frac{1}{1-\delta}))/(1-w\delta)) = 0
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   WolframAlpha will generate two possible solutions:

$$w^* = \frac{1-\sqrt{1-\delta^2}}{\delta} \text{ and } w^* = \frac{1+\sqrt{1-\delta^2}}{\delta}$$

   Which one looks more reasonable? Why?

   Solution: Let's see what happens for a reasonable δ like $\delta = 1/2$. The first would tell us the solution is:

$$w^*(1/2) = \frac{1-\sqrt{1-(1/2)^2}}{(1/2)} \approx 0.27 \qquad w^*(1/2) = \frac{1+\sqrt{1-(1/2)^2}}{(1/2)} \approx 3.7$$

   Obviously a cutoff of 3.7 would mean you never accept a job. The first one makes a lot more sense. It's so low because $\delta = 1/2$ is pretty impatient.

(d) Plot the solution on www.wolframalpha.com by plugging in the code below:

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plot w = (1-sqrt(1-\delta^2))/\delta from \delta=0 to \delta=1
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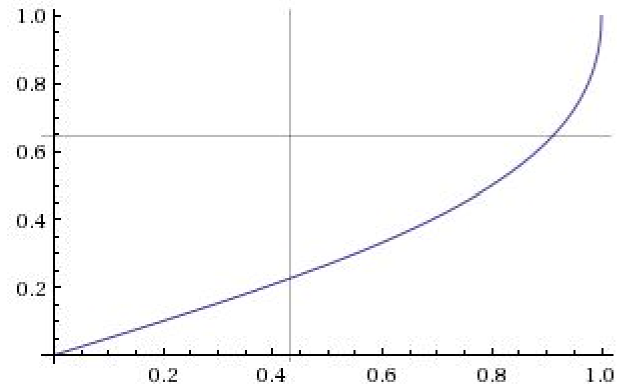
What happens to the optimal cutoff as $\delta \to 1$? Why?

Solution:



As you can see, the optimal cutoff goes to 1 as $\delta \to 1$. This makes sense. If you're infinitely patient, then you're willing to wait a long time to get the really high wage offer.

2. (Conditional probabilities - the dating game)(*From Angner 4.41*) You are considering asking L out for a date but you are a little worried that L is dating somebody else. The probability that L is dating somebody else is 1/4. If L is dating somebody else, he/she is unlikely to accept your offer to go on a date: in fact, you think the probability is only 1/6. If L is not dating somebody else, you think the probability is 2/3.

   (a) What is the probability that L is dating somebody else but will accept your offer to go on a date anyway?

   Solution: Let S be the event that L is dating somebody else. Let Y be the event that L accepts your date. The information we have then is the following:

   $$P(S) = 1/4, \quad P(Y|S) = 1/6, \quad P(Y|\neg S) = 2/3.$$

   That's what we know. We want to know here is $P(Y \cap S) = P(S)P(Y|S) = 1/24$.

   (b) What is the probability that L is not dating somebody else and will accept your offer to go on a date?

   Solution: $P(\neg S \cap Y) = P(\neg S)P(Y|\neg S) = 3/4 \cdot 2/3 = 1/2$.

   (c) What is the probability that L will accept your offer to go on a date?

   Solution: $P(Y) = P(Y|S)P(S) + P(Y|\neg S)P(\neg S) = 1/6 \cdot 1/4 + 2/3 \cdot 3/4 = 13/24$.

   (d) Suppose L accepts your offer to go on a date. What's the probability that L is dating somebody else, given that L agreed to go on a date?

   Solution: $P(S|Y) = \frac{P(S \cap Y)}{P(Y)} = \frac{P(Y|S)P(S)}{P(Y|S)P(S)+P(Y|\neg S)P(\neg S)} = \frac{1/6 \cdot 1/4}{1/6 \cdot 1/4 + 2/3 \cdot 3/4} = \frac{1}{13}$.

3. (Bayes rule and base rate neglect)(*Modified from Angner 5.13*) Doctors often encourage women over a certain age to participate in routing mammogram screening for breast cancer. Suppose that from past statistics, the following is known. At any one time, 1 percent of women have breast cancer. The test administered is correct in 90 percent of cases. That is, if the woman does have cancer, there is a 90 percent chance the result is positive. If she does not have cancer, there is a 90 percent chance the result is negative.

   (a) Given that a woman has a positive result, what is the probability she has breast cancer?

   Solution:

$$P(C|+) = \frac{P(+|C)P(C)}{P(+|C)P(C) + P(+|N)P(N)} = \frac{0.9 \cdot 0.01}{0.9 \cdot 0.01 + 0.1 \cdot 0.99} = \frac{1}{12}$$

   (b) If you ask a "person on the street" the question above, what might you expect that person's answer to be?

   Solution: People tend to answer 90%, misunderstanding what a test being 90% accurate means.

   (c) What is the *base-rate* here? Describe *base rate neglect.*

   Solution: The base rate is that original rate that 99% on women don't have cancer. That information is just as pertinent as the test result (the evidence), yet people often fail to factor that in. That's why we say there is *base rate neglect.* We also sometimes call that the *base rate fallacy.*

4. (Probability and private jets)(*Modified from Angner 5.12*) You earned so much money in Q1 that you can now afford a private jet. You have to decide between a few different jets that have different engine configurations. Use p to denote the probability that an engine fails during any one flight. A "catastrophic engine failure" is an engine failure that makes the plane unable to fly. Assume engine failures are independent events.

   (a) Jet A has just one engine (i.e. it's catastrophic if that engine fails). What is the probability of a catastrophic engine failure?

   Solution: p. Yeah – it's not a trick question.

   (b) Jet B has two engines and needs both of them to fly. What is the probability of a catastrophic engine failure? Is the twin-engine Jet B safer than single-engine Jet A?

   Solution: If you said $2p$, that's naughty (and wrong!). What if $p = 0.75$, then you have the probability of a catastrophic failure as 1.5, which isn't even a probability. OK, hopefully you didn't say $2p$. Since they're independent events, the probability that neither fails is $(1-p)(1-p)$. Then the probability that at least one fails (leading to a catastrophe is the complementary probability $1-(1-p)(1-p) = 2p - p^2$. Note that $2p - p^2 > p$ for $p \in (0,1)$, so Jet B is more dangerous.

   (c) Jet C has two engines and needs just one of them to fly. What is the probability of a catastrophic engine failure?

   Solution: Here the probability of a catastrophic failure is just the probability that both fail, which equals p^2. This is the safest plane yet discussed, as $p^2 < p < 2p - p^2$ for $p \in (0,1)$.

(d) Jet D has four engines and can fly on any two of them. What is the probability Jet D has a catastrophic failure?

Solution: Now the jet goes down if 3 or 4 engines fail. The probability of a specific three engines failing is $p^3(1-p)$. But there are four possible variants (varying the one not failing each time). Also, we need to add on the probability that all four fail:

$$\Pr(\text{Catastrophe}) = 4p^3(1-p) + p^4 = 4p^3 - 3p^4$$

(e) Jet E has four engines (two on each wing) and can fly as long as at least one engine on each wing is functional. What is the probability Jet D has a catastrophic failure?

Solution: This one's a bit tricky. Note that the plane fails if both engines on either wing fails. The probability of both engines failing on a given wing is p^2. Then the probability a given wing doesn't fail is $1-p^2$. Then the probability that neither wing fails is $(1-p^2)^2$. Then the probability that at east one wing fails (i.e. a catastrophe) is $1-(1-p^2)^2 = 2p^2 - p^4$.

If you disagree with any of the answers to this question, discuss it with me. I think I'm correct here, but there are a few ways to do each of these and they're easy to mess up!

5. (Roulette)(*Modified from Angner 6.9*) A roulette wheel has slots numbered $0, 00, 1, 2, \ldots, 36$. If you bet one dollar on a particular number, the casino usually pays you \$36 if the ball ends up in that slot. Thinking as a behavioral economist, why is there a 0 and a 00? Why not just number the slots from 1 to 38 instead?

Solution: Since the highest number is 36, unobservant (or stupid) folk will think the probability of any number hitting is 1/36, in which case the payout of 36-to-1 is actuarially fair. Of course, it's actually 1/38 given the 0 and 00, meaning that the casino is profiting in expectation.