ECON 455, Discussion Section 4

TA: Shane Auerbach (sauerbach@wisc.edu) ; Date: 02/20/15
Office: SS 6470. OH: Wed 8:00-9:30am; Thu 4:15-5:45pm; or by appt.

1. (Experiment) The point of these instructions is in the solutions so as not to bias the experiment.
   (a) Write a random sequence of twenty H's and T's, i.e. outcomes from twenty sequential fair coin tosses.
   (b) What is the most (and least) likely total number of T's and H's in the random sequence?
   (c) What is the most (and least) likely random sequence of T's and H's?
   (d) Other (more interesting) questions in the solutions, so as not to bias the experiment.

2. Don't forget the likelihood-ratio formulation of Bayes' Rules:

$$\frac{P(A|n)}{P(B|n)} = \frac{P(A)}{P(B)} \frac{P(n|A)}{P(n|B)}$$

   In words, we can say that the posterior equals the prior multiplied by the signal. Recall the model introduced in the lectures. A professor is trying to decide if a student is good or bad. The prior belief that the student is good is $1/2$. Then the professor receives signals (tests), where g represents a good test performance and b represents a bad test performance. Once a professor believes it more than 50% likely that a student is good (bad), she will misinterpret a bad (good) test performance as good (bad) with probability q. Also good students do good on tests and bad students do bad on tests with $P(g|G) = P(b|B) = p$.
   (a) Suppose $q = 1/2$ and $p = 3/4$. Given a naive professor observes $\tilde{g}\tilde{g}\tilde{g}$, what is her posterior probability that the student is good?
   (b) Now do the same exercise for a sophisticated professor.
   (c) Suppose the sophisticated professor's prior probability that the student is good is $\frac{50}{100}$. What is her posterior given she observes $\tilde{g}\tilde{g}$?
   (d) Suppose the sophisticated professor's prior probability that the student is good is $\frac{51}{100}$. What is her posterior given she observes $\tilde{g}\tilde{g}$?
   (e) Compare your answers to (c) and (d). What is counter-intuitive here? Where is it coming from in the model?
   (f) What is the professor's posterior if it is equally likely that good students and bad students do well on tests and she has no bias? Answer in words, appealing to the likelihood-ratio formulation of Bayes' Rule above.

3. (Mandatory drug testing)(Angner 5.28) In July 2011, the state of Florida started testing all welfare recipients for the use of illegal drugs. Statistics suggest that some 8 percent of adult Floridians use illegal drugs; let us assume that this is true for welfare recipients as well. Imagine that the drug test is 90 percent accurate, meaning that it gives the correct response in nine cases out of ten.

   (a) What is the probability that a randomly selected Floridian welfare recipient uses illegal drugs and has a positive test?

   (b) What is the probability that a randomly selected Floridian welfare recipient does not use illegal drugs but nevertheless has a positive test?

   (c) What is the probability that a randomly selected Floridian welfare recipient has a positive test?

   (d) Given that a randomly selected Floridian welfare recipient has a positive test, what is the probability that he or she uses illegal drugs?

   (e) If a Florida voter favors the law because he thinks the answer to (d) is in the neighborhood of 90 percent, what fallacy might he be committing?