ECON 455, Discussion Section 5

TA: Shane Auerbach (sauerbach@wisc.edu) ; Date: 02/27/15

Office: SS 6470. OH: Wed 8:00-9:30am; Thu 4:15-5:45pm; or by appt.

1. (Cutoff strategies) A risk-neutral decision-maker (DM) has a delicious Twinkie, the eating of which will immediately generate 1 util. There is also a magical dessert genie who shows up every period with a dessert of quality q_t where each q_t is an i.i.d. random draw from $\left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right\}$, with probability 1/3 on each possibility. In each period, the DM can eat either the dessert genie's offering (generating q_t utils), or the Twinkie (provided she has not already eaten it). The deliciousness continues infinitely into the future (and the Twinkie lasts forever, obviously), and the DM has discount rate δ.

   Warning: You can theoretically do this all by hand, but it will destroy you. I recommend using WolframAlpha (WA) for the algebra, and I use it in the solutions.

   (a) Let V^e denote the expected present value of entering a period having already eaten the Twinkie. Calculate V^e.

   (b) Let V_b^u denote the expected present value of entering a period with the Twinkie still uneaten/available given that the DM will eat the genie's offering if $q_t \in \{\frac{1}{2}, \frac{3}{4}\}$ but will eat the Twinkie instead if $q_t = \frac{1}{4}$. Calculate V_b^u.

   (c) Let V_c^u denote the expected present value of entering a period with the Twinkie still uneaten/available given that the DM will eat the genie's offering if $q_t = \frac{3}{4}$ but will eat the Twinkie instead if $q_t \in \left\{\frac{1}{4}, \frac{1}{2}\right\}$. Calculate V_c^u.

   (d) Let V_d^u denote the expected present value of entering a period with the Twinkie still uneaten/available given that the DM will eat the Twinkie no matter what. Calculate V_d^u.

   (e) Supposing $\delta = 3/4$, what is the DM's optimal cutoff strategy?

   (f) Find the optimal cutoff strategy for every $\delta \in (0, 1)$. To do this, compare V_b^u, V_c^u and V_d^u to derive conditions on δ under which each is the largest of the three. Try working with WolframAlpha to see if you can get it to do the work for you.

   (g) Explain the intuition behind your answer to (f).

   (h) Now suppose that the dessert genie, in each period, brings the DM a dessert randomly drawn from the continuous uniform distribution over $[0, 1]$. Let ϕ denote the DM's cutoff – for $q_t \leq \phi$, he eats the Twinkie while for $q_t > \phi$, he saves the Twinkie. Conjecture as to what you expect the optimal cutoff ϕ to be when $\delta = 0$ and also when $\delta = 1$, and explain your reasoning.

   (i) (Pretty hard!) Solve for the optimal cutoff, $\phi^*(\delta)$, i.e. that which maximizes V_ϕ^u for $\delta \in [0, 1]$. Compare your solution to the conjecture – graphing the solution is one nice way to compare.

2. (Deal or no deal, modified from Angner 6.13 and 6.41) You are on the show *Deal or No Deal*, where you are facing some boxes, each of which contains an unknown amount of money. At this stage, you are facing three boxes. One of them contains \$900,000, one contains \$300,000, and one contains \$60, but you do not know which is which. If you choose to open the boxes, you can open them in any order you like, but you can keep the amount contained in the last box only. Assume you have no outside, pre-existing wealth.

Warning: This question is very easy. Don't expect exams to be this easy.

   (a) What is the expected value of opening the three boxes?

   (b) The host gives you the choice between a sure \$400,000 and the right to open the three boxes. Assuming you want to maximize expected value, which should you choose?

   (c) You decline the \$400,000 and open a box. Unfortunately, it contains \$900,000. What is the expected value of opening the remaining two boxes?

   (d) The host gives you the choice between a sure \$155,000 and the right to open the remaining two boxes. Assuming you want to maximize expected value, what should you choose?

   (e) For the remaining parts, assume there are actually three boxes with \$1,000,000, \$1,000 and \$10 respectively and the host is offering you \$250,000 to stop playing. Assuming that you want to maximize expected value, would you accept the offer?

   (f) Assuming your utility function is $u(x) = \sqrt{x}$, and that you want to maximize expected utility, would you accept the offer?

   (g) Given the utility function above, what is the lowest amount would you accept to forego your right to continue opening the boxes?

3. (Angner 7.7) A person's value function is $v(x) = \sqrt{\frac{x}{2}}$ for gains and $v(x) = -2\sqrt{|x|}$ for losses. The person is facing the choice between a sure \$2 and a 50-50 gamble that pays \$4 if she wins and \$0 if she loses.

   (a) Show algebraically that this person is loss averse, in the sense that she suffers more when she loses \$4 than she benefits when she receives \$4.

   (b) If the outcomes are coded as gains, meaning that she will take the worst possible outcome as her reference point, what is the value of

      i. the sure amount?

      ii. the gamble?

      iii. Which would she prefer?

   (c) If the outcomes are coded as losses, meaning that she will take the best possible outcome as her reference point, what is the value of

      i. the sure amount?

      ii. the gamble?

      iii. Which would she prefer?