ECON 455, Discussion Section 6

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Office: SS 6470. OH: Wed 8:00-9:30am; Thu 4:15-5:45pm; or by appt.

Office hours will not be as above during midterm week. See email.

1. Jimmy wants to put an offer on a house. The house is worth \$200000 to him. He knows the market pretty well and knows that he is guaranteed to win if he submits an offer of \$150000 or above. If he bids $b \in [0, 150000]$, he will win with probability $b/150000$. If he wins, he pays his bid. Assume he has no outside wealth, so that his assets, x, are worth $200000 - b$ if he wins and zero if he loses.

   (a) Suppose Jimmy is an expected utility maximizer with $u(x) = \sqrt{x}$. Is Jimmy risk-averse, risk-loving or risk-neutral? What is his optimal bid?

   Solution: Jimmy is risk-averse because $u(x) = \sqrt{x}$ is concave. He solves the following problem:

$$\begin{aligned} &\max_b P(\text{He wins}|b)u(200000-b) + P(\text{He loses}|b)u(0) \\ =&\max_b \frac{b}{150000}u(200000-b) \\ =&\max_b \frac{b}{150000}\sqrt{200000-b} \end{aligned}$$

   Now we take a first order condition. This derivative is a little hard, but we can do it with the product rule ($d(f \cdot g) = df \cdot g + dg \cdot f$).

$$\frac{d\left(\frac{b}{150000}\sqrt{200000-b}\right)}{db} = \frac{\sqrt{200000-b}}{150000} - \frac{b}{150000}\frac{1}{2\sqrt{200000-b}}$$

   Setting equal to zero and rearranging:

$$\begin{aligned} \frac{\sqrt{200000-b}}{150000} - \frac{b}{150000}\frac{1}{2\sqrt{200000-b}} &= 0 \\ \frac{\sqrt{200000-b}}{150000} &= \frac{b}{150000}\frac{1}{2\sqrt{200000-b}} \\ 2(200000-b) &= b \\ 400000 &= 3b \\ b &= 400000/3 \approx 133333 \end{aligned}$$

   (b) Suppose Jimmy is an expected utility maximizer with $u(x) = x^2$. Is Jummy risk-neutral, risk-loving or risk-neutral? What is his optimal bid? Explain why the answer here is lower than the answer in (a).

   Solution: He is risk-loving because $u(x) = x^2$ is convex. He solves:

$$\max_b \frac{b}{150000}(200000-b)^2$$

   Taking the derivative (again with the product rule):

$$\frac{d\left(\frac{b}{150000}(200000-b)^2\right)}{db} = \frac{(200000-b)^2}{150000} - \frac{2b(200000-b)}{150000}$$

Setting it equal to zero gives us:

$$\frac{(200000-b)^2}{150000} - \frac{2b(200000-b)}{150000} = 0$$
$$\frac{(200000-b)^2}{150000} = \frac{2b(200000-b)}{150000}$$
$$200000 - b = 2b$$
$$b = \frac{200000}{3} \approx 66667$$

The answer here is lower because a lower bid in a sense represents a bigger gamble. That is, it lowers the probability you win but increases the payoffs in the event in which you win. Since Jimmy in (b) is risk-loving, he prefers a larger gamble than that undertaken by Jimmy in (a).

(c) Now suppose that Jimmy is a prospective utility maximizer. His value for gains and losses x is $v(x) = \sqrt{x}$ if $x > 0$ and $v(x) = -\sqrt{-x}$ if $x < 0$. Suppose also that he has a linear probability-weighting function, i.e. $\Pi(p) = p$. If his reference point is that he purchases the house for \$150,000, then what is his optimal bid.

Solution: If he were to purchase the house for \$150,000, then he would get a utility of \$50,000, so this is the reference point in terms of utility. Now, if he wins the house for his bid b, he will gain $150000-b$ relative to his reference point. If he loses, however, he will lose 50000 relative to his reference point. Therefore, he solves the following problem.

$$\max_b \frac{b}{150000}\sqrt{150000-b} - \left(1 - \frac{b}{150000}\right)\sqrt{50000}$$

Taking the derivative with respect to b and setting it equal to zero gives us

$$\frac{\sqrt{150000-b}}{150000} + \frac{1}{300\sqrt{5}} = \frac{b}{300000\sqrt{150000-b}}$$

Solving this for b gives us:

$$b = \frac{100000}{9}\left(8 + \sqrt{10}\right) \approx \$124025$$

(d) What about if his reference point is that he purchases the house for \$0?

Solution: Now, given his reference point is that he buys it for zero, his reference utility is 200000. Therefore, if he wins with bid b, he will lose b utils relative to his reference point. If he loses, then he will lose 200000 utils relative to his reference. Therefore he solves the following problem:

$$\max_b -\frac{b}{150000}\sqrt{b} - \left(1 - \frac{b}{150000}\right)\sqrt{200000}$$

Taking the derivative with respect to b and setting equal to zero gives us

$$\frac{400\sqrt{5} - 3\sqrt{b}}{300000} = 0$$

Solving for b gives us

$$b = \frac{800000}{9} \approx \$88889$$

(e) Continue to assume his reference point is that he purchases the house for \$0, but now assume that his probability weighting function is $\pi(p) = p^2$. Solve for his optimal bid. (Warning: Math gets awful - use WolframAlpha or skip it!)

Solution: He solves:

$$\max_b -\left(\frac{b}{150000}\right)^2 \sqrt{b} - \left(1 - \frac{b}{150000}\right)^2 \sqrt{200000}$$

Setting the derivative equal to zero gives us

$$\frac{b^{\frac{3}{2}}}{9000000000} + \frac{\frac{b}{150000} - 1}{75\sqrt{5}} = 0$$

Solving for b gives something very ugly in exact form, but if we look at it numerically,

$$b \approx 83704$$

(f) Now take the approach of Cumulative Prospective Utility and rewrite the maximization problem from (e) in this framework.

Solution: Now, the worst payoff, call it x_1, is a loss of 200000. The second-to-worst (and the best) payoff, call it x_2 is a loss of b. $P_1 = p_1 + p_2$ is the probability you get a payoff at least as good as x_1 and $P_2 = p_2$ is the probability you get a payoff at least as good as x_2. Then, we solve the problem

$$\begin{aligned} PU &= v(x_1) + \pi(P_2)(v(x_2) - v(x_1)) \\ &= -\sqrt{200000} + \left(\frac{b}{150000}\right)^2 \left(-\sqrt{b} - \left(-\sqrt{200000}\right)\right) \end{aligned}$$

So that's what we would maximize.

2. Remaining time to be spent going over PS2.