ECON 455, Discussion Section 7

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Office: SS 6470. OH: Wed 8:00-9:30am; Thu 4:15-5:45pm; or by appt.

1. Write up the normal (strategic) form (i.e. a bi-matrix of payoffs) for *Rock, Paper, Scissors* given that a player gets one util if she wins, negative one if she loses, and zero otherwise. Is this game static (i.e. one-shot) or dynamic? To which canonical 2x2 game is *Rock, Paper, Scissors* most similar?

   Solution:

	r	p	s
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

   Rock, Paper, Scissors is obviously static – the whole point is that both players must decide simultaneously. It is most similar to *Matching Pennies* in that it is a game of pure conflict.

2. Silvio Berlusconi has just received some terrible news – he turns to you as his advisor in all things game theoretic. On a recent trip to Toronto, Berlusconi had a late night out with Toronto mayor Rob Ford and not-Toronto-mayor Justin Bieber. That's not the terrible news though - he had a great time! The terrible news is that one of Toronto's newspapers, *The Globe and Mail*, has photos of the three doing cocaine behind a Tim Horton's at a truckstop at 4am. *The Globe and Mail* will either release those photos or choose not to. Silvio must decide how to handle the situation simultaneously to the newspaper's decision on whether to print or not. The newspaper has no soul, unlike poor Silvio, and therefore gets a utility of zero no matter what happens.

   You and Silvio come up with four options:

   W: He can go to the other paper, *The Star*, and give them an exclusive on his night-out. If he does this, it doesn't matter whether *The Globe and Mail* prints the photos or not, because everybody will read the exclusive instead. This option would be embarrassing for him, yielding him a utility of -2.

   X: He can deny rumors of the night-out. If *The Globe and Mail* prints the photos, then he is revealed to be a liar and he gets a utility of -3. If the paper doesn't print them, however, he not only gets no negative utility, but he is so thrilled to get away with his lie that he gets utility of 1.

   Y: He can attempt to distract the public by having his Italian media outlets publish sensational stories accusing footballer Mario Balotelli of being a unicorn sent from an alien unicorn planet. This would make his media outlets seem a little silly, but would lessen the impact of the photos, if published. His utility if the photos are published would be -2. If the photos are not published, his utility would be zero.

   Z: He can buy *The Globe and Mail* and destroy the photos. Newspapers are worth nothing nowadays, and he has plenty of money. If he buys the paper, it doesn't matter what decision the paper makes - he'll destroy the photos before they could go to print - he gets a utility of -1 whatever the paper decides.

   Silvio asks you to provide the following:

   (a) A formal description of the game, $G = \langle I, (A_i, u_i)_{i \in I} \rangle$.

(b) A representation of the game in normal/strategic form.

(c) A graph of the expected utility of each of Silvio's action as a function of his conjecture – that is, plot $u_S(a_s, \mu^S)$ for each $a_s \in A_s$. You can plot them all on one graph.

(d) A recommendation on actions above that Silvio should definitely avoid.

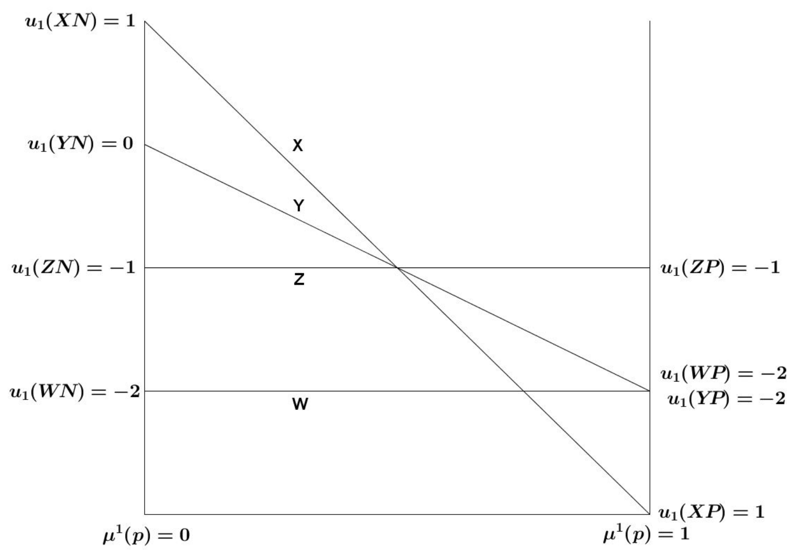
(e) A recommendation for Silvio given that he thinks the probability that *The Globe and Mail* publishes the photos is 40%.

Solution:

The set of players is $I = \{$Silvio, *The Globe and Mail*$\}$. The set of actions for Silvio is $A_1 = \{W, X, Y, Z\}$ as defined above. The set of actions for *The Globe and Mail* is $A_2 = \{N, P\}$, where N represents not printing the photos and P represents printing them. The utility functions are $u_1(WN) = u_1(WP) = -2, u_1(XN) = 1, u_1(XP) = -3, u_1(YN) = 0, u_1(YP) = -2, u_1(ZN) = u_1(ZP) = -1$. For the newspaper, $u_2(\cdot) = 0$ for all possible outcomes. The normal/strategic form of the game is therefore as follows.

	N	P
W	-2,0	-2,0
X	1,0	-3,0
Y	0,0	-2,0
Z	-1,0	-1,0

The expected payoffs as a function of conjectures should look like the following.



As for what Silvio should avoid, our first graph suggests that only W is not justifiable, in that it's not a best response for any conjecture. It might seem like Y is a bad choice, given that it's not the unique best response for any conjecture, but it is a best response (along with X and Z) when $\mu^1(p) = 1/2$.

Finally, given $\mu^1(p) = 0.4$, we can see from the graph that X is the best response.

3. (Osborne & Rubinstein Exercise 64.1) Each of two players announces a non-negative integer equal to at most 100. If $a_1 + a_2 \leq 100$, then each player i receivers a payoff of a_i. If $a_1 + a_2 > 100$, and $a_i < a_j$, then player i receives payoff of a_i and player j receives $100 - a_i$;

if $a_1 + a_2 > 100$ and $a_i = a_j$ then each player receives 50. Show that the game is dominance solvable and find the set of surviving outcomes. Here, you may eliminate weakly dominated strategies in solving the game, though this is dicey in general.

Solution:

$A_i = [0, 100] \cap \mathbb{Z}$. Any $a_i < 50$ is strictly dominated by $a_i = 50$:

a_{-i}	$u_i(a_i = 50)$	$u_i(a_i < 50)$
$a_{-i} > 50$	50	$a_i < 50$
$a_{-i} \leq 50$	50	$a_i < 50$

And 50 is weakly dominated by 51 and 100 is weakly dominated by 99 (check for yourself).

$$A = [0, 100] \cap \mathbb{Z} \times [0, 100] \cap \mathbb{Z}$$
$$ND^1(A) = [51, 99] \cap \mathbb{Z} \times [51, 99] \cap \mathbb{Z}$$

Now, 51 is the unique best response to 52, so 51 is not weakly dominated. However, given that 100 has been eliminated from consideration, 98 weakly dominates 99. Similarly, once 99 is eliminated from consideration, 97 weakly dominates 98. And this process continues until only 51 remains.

$$ND^2(A) = [51, 98] \cap \mathbb{Z} \times [51, 98] \cap \mathbb{Z}$$
$$ND^3(A) = [51, 97] \cap \mathbb{Z} \times [51, 97] \cap \mathbb{Z}$$
$$\cdots$$
$$ND^{49}(A) = \{51\} \times \{51\}$$

Since we got down to a single profile of actions, we can say that this game is dominance solvable (but we did need to use weak dominance).

4. (Osborne Exercise 372.9) Give an example of an action that is a best response to a belief (a *conjecture*, that is) but is not a best response to any belief that assigns probability 1 to a single action. One way to do this is to design your own static game such that this is true.

   Solution:

   Consider the following game. If $\mu^1(l) = \mu^1(r) = \frac{1}{2}$, then M is the best response, even though M is not a best response to either l or r.

   Only P1 Payoffs Shown :

	l	r
T	3	0
M	2	2
B	0	3

5. (Watson Exercise 6.1) Find the set of rationalizable actions for each player in **G** – note, rationalizable actions are those that survive iterated deletion of strictly dominated strategies.

   G :

	W	X	Y	Z
U	3,6	4,10	5,0	0,8
M	2,6	3,3	4,10	1,1
D	1,5	2,9	3,0	4,6

Solution:

To find the set of rationalizable strategies, we see what survives iterated deletion of strictly dominated strategies. First, note that X strictly dominates Z, so we can eliminate Z from consideration.

G :

	W	X	Y
U	3,6	4,10	5,0
M	2,6	3,3	4,10
D	1,5	2,9	3,0

Now note that M strictly dominates D, so we remove D from consideration. But actually U also strictly dominated both M and D, so we can eliminate both. And given that, the best response to U for P2 is X. Therefore, only U and X are rationalizable.

6. (Hotelling's Model of Electoral Competition) Suppose there are two candidates in an election and the set of possible positions they can take is $\{0, 1, 2, \ldots, l\}$, where l is even. Each voter has his own position, and will vote for whichever candidate is closest to his position. Suppose there exists a position m such that exactly half of the voters' positions are at least m and exactly half of the voters' positions are at most m. Each candidate wants to win the election by getting the most votes – they only care about winning, not how many votes they get. If the two candidates get the same number of votes, they both lose. What actions survive iterated deletion of weakly dominated strategies?

   Solution:

   This is actually not the best question, because you can argue immediately that choosing m as your position weakly dominates any other. So you don't need iteration.

	$u_i(a_i < m)$	$u_i(a_i = m)$	$u_i(a_i > m)$
$a_{-i} < m$	Win or Lose	Win	Win or Lose
$a_{-i} = m$	Lose	Lose	Lose
$a_{-i} > m$	Win or Lose	Win	Win or Lose

   From the chart above, you can see $a_i = m$ weakly dominates other strategies.