ECON 455, Discussion Section 9

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Office: SS 6470. OH: Wed 8:00-9:30am; Thu 4:15-5:45pm; or by appt.

1. (Self-justification; Lec 18) It will either rain, snow or be sunny today. The weather forecast tells you that the probability of each of these events is r, s and $(1 - r - s)$ respectively. You have to decide what to wear today, a decision that yields payoffs contingent on the weather as follows:

	Rain	Snow	Sun
Coat	10	0	10
Winter jacket	0	10	10
T-shirt	0	0	20

(a) For what r, s is ...

i. the coat optimal?

Solution: The coat is optimal when it yields higher expected payoffs than winter jacket and t-shirt, that is

$$10r + 10(1-r-s) \geq 10s + 10(1-r-s) \text{ and } 10r + 10(1-r-s) \geq 20(1-r-s)$$

That simplifies to $r > s$ and $2r \geq 1 - s$.

ii. the winter jacket optimal?

Solution: The winter jacket is optimal when it yields higher expected payoffs than winter jacket and t-shirt, that is

$$10s + 10(1-r-s) \geq 10r + 10(1-r-s) \text{ and } 10s + 10(1-r-s) \geq 20(1-r-s)$$

That simplifies to $s > r$ and $2s \geq 1 - r$.

iii. the t-shirt optimal?

Solution: The t-shirt is optimal when it yields higher expected payoffs than winter jacket and t-shirt, that is

$$20(1-r-s) \geq 10r + 10(1-r-s) \text{ and } 20(1-r-s) \geq 10s + 10(1-r-s)$$

That simplifies to $1 - r - s \geq r$ and $1 - r - s \geq s$.

(b) Suppose you have your own subjective beliefs that put probability $\hat{r}, \hat{s}$ and $(1 - \hat{r} - \hat{s})$ on rain, snow and sun respectively. Also suppose that you choose an action to maximize expected utility at beliefs $(\alpha r + (1-\alpha)\hat{r}, \alpha s + (1-\alpha)\hat{s}, \alpha(1-r-s) + (1-\alpha)(1-\hat{r}-\hat{s}))$, and you then distort those beliefs to maximize your expected utility, but only insofar as you can do so without making the action you chose suboptimal. This is "Model 3" from the lecture, and the one you should focus on. Remember that the algorithm for solving these is to first work out what subjective beliefs would justify each action. Then, for each action, find which subjective beliefs (anything so long as they justify the action) maximize the expected utility of that action. Then compare those maximal expected utilities to pick the action.

i. Supposing $\alpha = \frac{1}{2}, r = \frac{1}{2}, s = \frac{1}{4}$, find the action and the beliefs $\hat{r}, \hat{s}$ that you choose.

Solution: He chooses the action that maximizes expected utility at beliefs:

$$\left(\frac{1}{4}+\frac{1}{2}\hat{r}\ ,\ \frac{1}{8}+\frac{1}{2}\hat{s}\ ,\ \frac{1}{8}+\frac{1}{2}(1-\hat{r}-\hat{s})\right)$$

Therefore, he chooses ...

- Coat if the following two conditions are met. The first:

$$C \succeq W$$
$$10\left(\frac{1}{4}+\frac{1}{2}\hat{r}\right)+10\left(\frac{1}{8}+\frac{1}{2}(1-\hat{r}-\hat{s})\right) \geq 10\left(\frac{1}{8}+\frac{1}{2}\hat{s}\right)+10\left(\frac{1}{8}+\frac{1}{2}(1-\hat{r}-\hat{s})\right)$$
$$\hat{r} \geq \hat{s}-\frac{1}{4}$$

  The second:

$$C \succeq T$$
$$10\left(\frac{1}{4}+\frac{1}{2}\hat{r}\right)+10\left(\frac{1}{8}+\frac{1}{2}(1-\hat{r}-\hat{s})\right) \geq 20\left(\frac{1}{8}+\frac{1}{2}(1-\hat{r}-\hat{s})\right)$$
$$2\hat{r} \geq \frac{3}{4}-\hat{s}$$

- Winter jacket if the following two conditions are met. The first (opposite of $C \succeq W$ above):

$$W \succeq C$$
$$\hat{s} \geq \hat{r}+\frac{1}{4}$$

  The second:

$$W \succeq T$$
$$10\left(\frac{1}{8}+\frac{1}{2}\hat{s}\right)+10\left(\frac{1}{8}+\frac{1}{2}(1-\hat{r}-\hat{s})\right) \geq 20\left(\frac{1}{8}+\frac{1}{2}(1-\hat{r}-\hat{s})\right)$$
$$\hat{s} \geq 1-\hat{r}-\hat{s}$$

- T-shirt if the following two conditions are met. The first (opposite of $C \succeq T$ above):

$$T \succeq C$$
$$2\hat{r}+\hat{s} \leq \frac{3}{4}$$

  The second (opposite of $W \succeq T$ above):

$$T \succeq W$$
$$1-\hat{r}-\hat{s} \geq \hat{s}$$

Now we know the subjective beliefs that justify each action. The second step of the algorithm is to, for each action, find which subjective belief, out of those that justify that action, maximized the expected utility from that action. Formally, for each actions, this amounts to solving a constrained maximization problem over two variables, with the constraints being to ensure that we're only considering subjective beliefs that justify the given action.

However, the problem is actually a fair bit simpler than that. Consider the actions one-by-one:

- If you wear the coat, you don't care if it rains or if it's sunny, but you don't like when it snows. Therefore, optimal subjective beliefs will be those that set $\hat{s} = 0$. As for $\hat{r}$, you're indifferent as to whether it's rainy or sunny, so any $\hat{r} \in [0, 1]$ is optimal. However, we also need that $\hat{r}$ to be such that $\hat{r}, \hat{s}$ justify wearing the coat. In particular, that means that $2\hat{r} \geq \frac{3}{4} - \hat{s}$. Since $\hat{s} = 0, \hat{r} \geq \frac{3}{8}$. Then the expected utility in this case (here I'll set $\hat{r} = 1$, though it's the same for any $\hat{r}$):

$$\begin{aligned} EU &= 10\left(\frac{1}{4} + \frac{1}{2}\hat{r}\right) + 10\left(\frac{1}{8} + \frac{1}{2}(1 - \hat{r} - \hat{s})\right) \\ &= 10\left(\frac{1}{4} + \frac{1}{2}\right) + 10\left(\frac{1}{8}\right) \\ &= \frac{35}{4} \end{aligned}$$

- If you wear the winter jacket, you don't care if it snows or if it's sunny, but you don't like when it rains. Therefore, optimal subjective beliefs will be those that set $\hat{r} = 0$. As for $\hat{s}$, you're indifferent as to whether it's snowy or sunny, so any $\hat{s} \in [0, 1]$ is optimal. However, we also need that $\hat{s}$ to be such that $\hat{r}, \hat{s}$ justify wearing the coat. In particular, that means that $\hat{s} \geq 1 - \hat{r} - \hat{s}$. Since $\hat{r} = 0, \hat{s} \geq \frac{1}{2}$. Then the expected utility in this case (here I'll set $\hat{s} = 1$, though it's the same for any $\hat{s}$):

$$\begin{aligned} EU &= 10\left(\frac{1}{8} + \frac{1}{2}\hat{s}\right) + 10\left(\frac{1}{8} + \frac{1}{2}(1 - \hat{r} - \hat{s})\right) \\ &= 10\left(\frac{1}{8} + \frac{1}{2}\right) + 10\left(\frac{1}{8}\right) \\ &= \frac{15}{2} \end{aligned}$$

- If you wear the t-shirt, you only get utility if it's sunny. Therefore, optimal subjective beliefs will be those that set $\hat{r} = \hat{s} = 0$. Obviously the t-shirt is justifiable with $\hat{r} = \hat{s} = 0$.

$$\begin{aligned} EU &= 20\left(\frac{1}{8} + \frac{1}{2}(1 - \hat{r} - \hat{s})\right) \\ &= 20\left(\frac{1}{8} + \frac{1}{2}\right) \\ &= \frac{25}{2} \end{aligned}$$

Since $\frac{25}{2}$ is the largest of the three, the optimal action is to wear the t-shirt and the optimal beliefs are $\hat{s} = \hat{r} = 0$.

ii. Supposing $\alpha = 1, r = \frac{1}{2}, s = \frac{1}{4}$, find the action and the beliefs $\hat{r}, \hat{s}$ that you choose. *Hint: You do not need to solve this out. Instead, think what $\alpha = 1$ means and use your answer to answer to (a).*

Solution: When $\alpha = 1$, we simply take the objective probabilities as given, so we return to the case in (a). With $r = \frac{1}{2}$ and $s = \frac{1}{4}$, we see that the actions for which both conditions are satisfied is the coat, so we wear the coat. Technically we can choose $\hat{r}$ and $\hat{s}$ however we want. Since they affect neither our decisions nor our payoffs, they're irrelevant!

(c) Suppose instead we use "Model 1" from the lecture. In this case, you work out the what action is best for each set of subjective beliefs $(\hat{r}, \hat{s}, 1 - \hat{r} - \hat{s})$ and then pick the beliefs to maximize your expected utility given those believes. What is the optimal $(\hat{r}, \hat{s}, 1 - \hat{r} - \hat{s})$ and action? How does this illustrate the main criticism of "Model 1"? *Hint: To find the optimal action and belief, all you really need to do is look at the payoff matrix and think about it for a moment.*

The maximum payoff you can get is 20. This can be your expected payoff if you just convince yourself that $\hat{r} = \hat{s} = 0$ and choose to wear a t-shirt, so that's the optimal action and set of beliefs. This clearly illustrates the criticisms of Model 1: your beliefs need not have anything to do with the objective probabilities of the events, so you always just choose the action with the best possible payoff and then set your beliefs such that you get that maximum payoff with probability one.

2. (Self-justification; Model 2 MC) Which of the following best describes "Model 2" (and its relation to the other models) of self-justification in Lecture 18?

(a) Unlike "Model 1," the true probabilities factor into the agent's decision. Unlike "Model 3," the agent chooses the optimal action based only upon the true probabilities, then chooses the subjective beliefs to maximize expected utility given the chosen action.

(b) Unlike "Model 1," the true probabilities factor into the agent's decision. Unlike "Model 3," an agent's chosen action may not be optimal for his chosen subjective beliefs.

(c) Unlike "Model 1," the agent chooses his beliefs before choosing his action. Unlike "Model 3," the agent chooses the optimal action based only upon the true probabilities, then chooses the subjective beliefs to maximize expected utility given the chosen action.

(d) Unlike "Model 1," the agent chooses his beliefs before choosing his action. Unlike "Model 3," an agent's chosen action may not be optimal for his chosen subjective beliefs.

Solution: There are four statements to consider (which are combined in various ways across the four answer choices):

- Unlike "Model 1," the true probabilities factor into the agent's decision.
- Unlike "Model 1," the agent chooses his beliefs before choosing his action.
- Unlike "Model 3," the agent chooses the optimal action based only upon the true probabilities, then chooses the subjective beliefs to maximize expected utility given the chosen action.
- Unlike "Model 3," an agent's chosen action may not be optimal for his chosen subjective beliefs.

The first statement is true. The agent puts weight α on the true probabilities in making his decision. Note that this is also true of Model 3. The second statement is false. In all of the models, you can think of the agent choosing beliefs and actions simultaneously. You can also think of him choosing the action first then choosing the beliefs subsequently, but his choice of action anticipates the choice of beliefs he will make thereafter. The third statement is false – in both Model 2 and Model 3, the agent picks his action based upon an $(\alpha, 1-\alpha)$ weighting over the true and subjective probabilities, not just on the true probabilities. The fourth statement is true. If you remember the umbrella example, you might remember that the agent would sometimes choose to bring the umbrella and set his subjective belief to a zero percent chance of rain – obviously then bringing the umbrella is inconsistent with the zero percent chance of rain. Since statements 1 and 4 are true and 2 and 3 are fase, the correct answer is (b).

3. (Self-deception; Lec 19) Which of the following was NOT part of the circumstance we found in class in which you could want to avoid finding out free information about yourself?
   (a) You are over-confident.
   (b) You are a quasi-hyperbolic discounter.
   (c) If you do not see the information, you will do the task.
   (d) It is optimal to not do the task.
   (e) You decide whether to see the information in the period before that in which you decide whether to undertake a certain task.

   Solution: In the model we looked at, to be over-confident (a) was equivalent to saying that if you did not see the information, you would do the task (c), and both of those were key elements in the case in which it could be smart to avoid the information. (b) was also critical, because we had to want $t = 0$ self to want the $t = 1$ self to do the task even when the $t = 1$ self had information that made him not want to take on the task. If you ever selfs of different periods in conflict, that's a sign that you need non-standard discounting, and that's what we had here. (e) is also critical. If you could decide on whether to see the information in the same period that you make the effort choice, you would always want to see the info. (d), however, was not part of the model – usually it was optimal to do the task. So the answer is (d).

4. (Self-handicapping; Lec 19) Based on our model of self-handicapping, why might you drink (alcohol, that is) before an important exam?
   (a) Alcohol may calm you down and therefore make it less likely that you self-handicap during the exam.
   (b) You want to make the test result less informative.
   (c) If you do really well on an exam after drinking beforehand, you get super confident.
   (d) Because it's senior spring and your grade doesn't really matter.
   (e) Because the Badgers lost. And because the referees kinda screwed us.
   (f) Same reason as why Shane drinks before/while writing discussion section handouts.

Solution: The answer is of course (b). You're making the test a less informative signal of your knowledge. To see how, recall our likelihood-ratio version of Bayes' rule:

$$\frac{P(G|b)}{P(B|b)} = \frac{P(G)}{P(B)} \frac{P(b|G)}{P(b|B)}$$

If we drink before the test, then $P(b|G)$ moves closer to $P(b|B)$ (i.e. gets higher). That is, the likelihood that we do bad on the test given that we're actually smart is closer to the likelihood that we do poorly on the test given that we're bad because, although we may be actually G (knowledgeable), we were also drunk... In doing so, our posterior will actually move less given a bad test result than it would have had we got that same bad test result and not handicapped ourself beforehand. So handicapping ourselves can leave us with more favorable posteriors. (d), (e) and (f) may also be acceptable answers – depends who's grading it.