NU Econ 101: Lecture 4

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Where are we now?

Acronyms

I use a few acronyms in these slides:

- Curves and lines:
  - IC: Indifference curve
  - BC: Budget constraint
- Types of utility functions:
  - CD: Cobb-Douglas utility function
  - Log: Logarithmic utility function
  - PC: Perfect complements utility function
  - PS: Perfect substitutes utility function
- Effects:
  - IE: Income effect
  - SE: Substitution effect

Where are we now?

Choice

What we know:

- How to model preferences.
- How to model the budget constraint.

What we want to know:

- How agents *choose* consumption bundles.

Combining BC and IC (1)

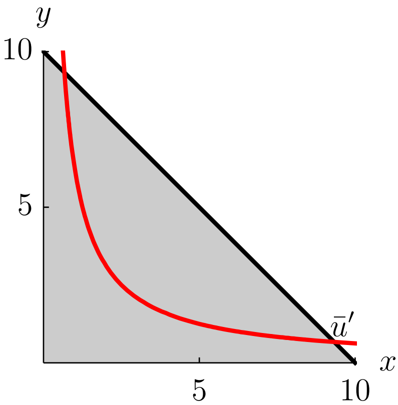
Let's take a very simple example with logarithmic utility:

$$u(x, y) = \ln(x) + \ln(y).$$

And suppose $p_x = p_y = 1$ and $w = 10$.

We will now plot the budget constraint and a few indifference curves together ...

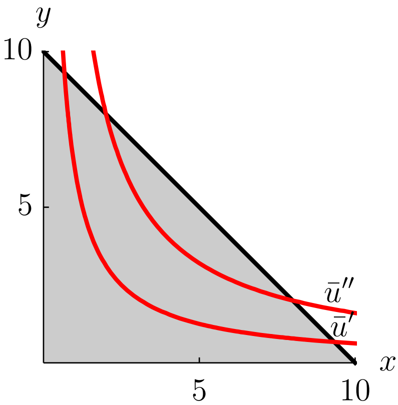
Combining BC and IC (2)



Can we do better than $\bar{u}'$?

- Yes! Anything between the lines is better.

Combining BC and IC (3)

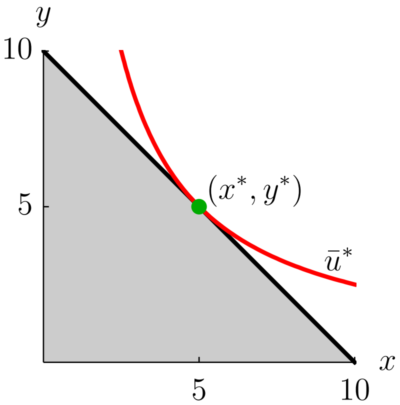


Can we do better than $\bar{u}''$?

- Yes! Anything between $\bar{u}''$ and the BC is better.

Combining BC and IC (4)

We continue this logic until we get ...



We find that $(x^*, y^*) = (5, 5)$ is the optimal bundle.

Characterizing the optimal bundle

We don't want to have to always draw the budget constraint and indifference curves until we get the right one.

- And we do not know which $\bar{u}$ touches perfectly.

What two things can we say about the optimal bundle?

1. It is on the budget constraint.
2. The slope of the indifference curve running through it matches the slope of the budget constraint.

Combine these two and we have **the golden rule for optimal consumption**.

Solving for the optimal bundle (1)

Let's try it!

First, the optimal bundle is on the budget constraint, so

$$\begin{aligned} p_x \cdot x^* + p_y \cdot y^* &= w \\ 1 \cdot x^* + 1 \cdot y^* &= 10 \\ x^* + y^* &= 10 \\ y^* &= 10 - x^*. \end{aligned}$$

That gives us one of the two equations we need to solve for the optimal bundle.

Solving for the optimal bundle (2)

Second, the slope of the indifference curve running through the optimal bundle matches the slope of the budget constraint.

- The slope of the budget constraint is $-\frac{p_x}{p_y}$.
- The slope of the IC through (x^*, y^*) is $-MRS$.

Our second equation, then, is

$$MRS_{xy}(x^*, y^*) = \frac{p_x}{p_y}$$

$$\frac{MU_x(x^*, y^*)}{MU_y(x^*, y^*)} = \frac{p_x}{p_y}$$

$$\Longrightarrow$$

$$\frac{1/x^*}{1/y^*} = \frac{1}{1}$$

$$\frac{y^*}{x^*} = 1$$

$$y^* = x^*$$

Solving for the optimal bundle (3)

Now we just have to combine our two equations:

1. On BC: $y^* = 10 - x^*$
2. Slopes match: $y^* = x^*$

Plug the second into the first and we get $x^* = 10 - x^*$ which solves to $x^* = 5$. Then plug that into either to get $y^* = 5$:

$$(x^*, y^*) = (5, 5).$$

It works!

An example with Cobb-Douglas (1)

It works the same way with Cobb-Douglas.

Suppose this time that:

- $p_x = 3$ and $p_y = 6$
- $w = 18$
- $u(x, y) = \sqrt{x} \cdot \sqrt{y}$

Note also the derivative we will need:

$$\frac{d}{dx}(\sqrt{x} \cdot \sqrt{y}) = \frac{\sqrt{y}}{2\sqrt{x}}.$$

An example with Cobb-Douglas (2)

First, the optimal bundle is on the budget constraint, so

$$p_x \cdot x^* + p_y \cdot y^* = w$$
$$3 \cdot x^* + 6 \cdot y^* = 18$$
$$y^* = 3 - \frac{1}{2}x^*.$$

One equation down, one to go . . .

An example with Cobb-Douglas (3)

Second, the slope of the indifference curve running through the optimal bundle matches the slope of the budget constraint:

$$MRS_{xy}(x^*, y^*) = \frac{p_x}{p_y}$$

$$\frac{MU_x(x^*, y^*)}{MU_y(x^*, y^*)} = \frac{p_x}{p_y}$$

$$\frac{\left(\frac{\sqrt{y}}{2\sqrt{x}}\right)}{\left(\frac{\sqrt{x}}{2\sqrt{y}}\right)} = \frac{3}{6}$$

$$\Longrightarrow$$

$$\left(\frac{\sqrt{y}}{2\sqrt{x}}\right) \cdot \left(\frac{2\sqrt{y}}{\sqrt{x}}\right) = \frac{1}{2}$$

$$\frac{y^*}{x^*} = \frac{1}{2}$$

$$y^* = x^*/2$$

An example with Cobb-Douglas (4)

Now we just have to combine our two equations:

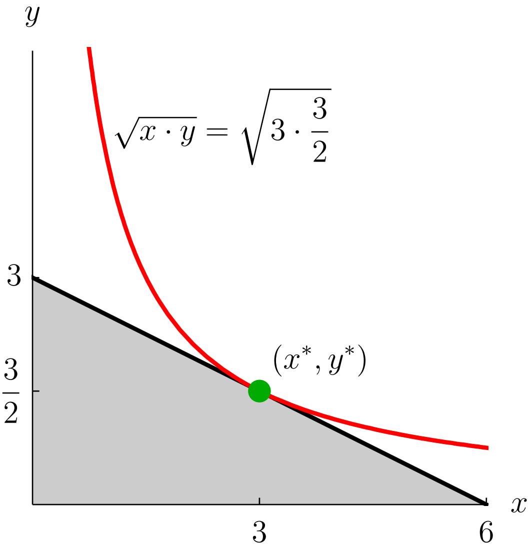
1. On BC: $y^* = 3 - \frac{1}{2}x^*$
2. Slopes match: $y^* = x^*/2$

Plug the second into the first and we get $x^*/2 = 3 - x^*/2$ which solves to $x^* = 3$. Then plug that into either to get $y^* = 3/2$:

$$(x^*, y^*) = \left(3, \frac{3}{2}\right).$$

It works! Now let's have a look . . .

An example with Cobb-Douglas (5)



Notes on the golden rule (1)

Note 1:

Why can we assume that it is optimal to spend all of your money?

- Because of the *free disposal assumption*.
  - Fixing y, if $x' > x$, $u(x', y) \geq u(x, y)$.
- Because the goods are perfectly indivisible.
  - With indivisibilities, may be impossible to spend all of w.

Notes on the golden rule (2)

Note 2:

The second equation of the golden rule,

$$MRS_{xy}(x^*, y^*) = \frac{p_x}{p_y},$$

has an economic interpretation:

- At the optimum consumption bundle, your relative values for the two goods (your MRS) is equal to the market's relative values for the two goods (p_x/p_y).
  - Makes sense – if the relative values were different, you and the market could find a trade that benefits both.

Notes on the golden rule (3)

Note 3:

We can rearrange the equation that matches the slopes as follows:

$$MRS_{xy}(x^*, y^*) = \frac{p_x}{p_y}$$
$$\frac{MU_x(x^*, y^*)}{MU_y(x^*, y^*)} = \frac{p_x}{p_y}$$
$$\frac{MU_x(x^*, y^*)}{p_x} = \frac{MU_y(x^*, y^*)}{p_y}$$

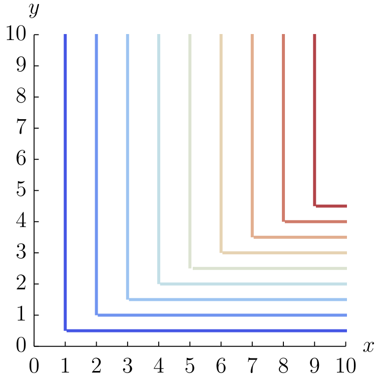
In the final line, we see that equating the slopes of the indifference curve and the budget constraint is equivalent to equating each good's marginal utility divided by its price (i.e. its **bang per buck**).

Where are we now?

The problem with perfect complements

We cannot use **the golden rule** as is for perfect complements.

- Why? Recall the indifference curves ...



- The slope is either $-\infty$ or 0.
- Cannot equate it with p_x/p_y.

Perfect complements graphically (1)

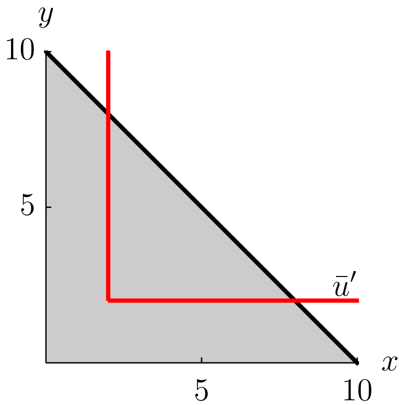
Let's take a very simple example with perfect complements:

$$u(x, y) = \min(x, y).$$

And suppose $p_x = p_y = 1$ and $w = 10$.

We will now plot the budget constraint and a few indifference curves together . . .

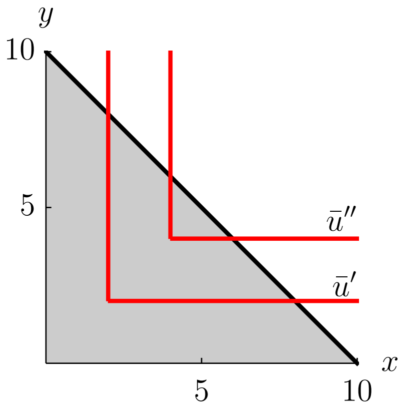
Perfect complements graphically (2)



Can we do better than $\bar{u}'$?

- Yes! Anything between the lines is better.

Perfect complements graphically (3)

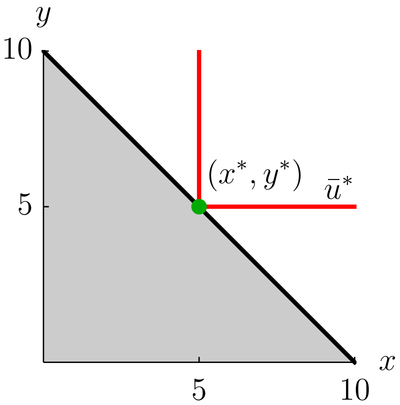


Can we do better than $\bar{u}''$?

- Yes! Anything between $\bar{u}''$ and the BC is better.

Perfect complements graphically (4)

We continue this logic until we get . . .



We find that $(x^*, y^*) = (5, 5)$ is the optimal bundle.

Characterizing the optimal bundle (PC)

First equation: It's still true that the optimal bundle will always be on the budget constraint, so we still have $p_x \cdot x^* + p_y \cdot y^* = w$.

Second equation: The slopes of the IC and BC will not equate. Instead, note that optimal bundle always at *the kink*, i.e., where the goods are consumed at the optimal ratio. So, for general perfect complements preferences,

$$u(x, y) = \min(a \cdot x, b \cdot y),$$

with a, b two positive numbers, use equation:

$$a \cdot x^* = b \cdot y^*.$$

An example with perfect complements (1)

Suppose this time that:

- $p_x = 2$ and $p_y = 3$
- $w = 30$
- $u(x, y) = \min(2x, 3y)$

Quick check: which is correct?

1. We want 2 units of x for every 3 units of y.
2. We want 3 units of x for every 2 units of y. ✓

An example with perfect complements (2)

First, the optimal bundle is on the budget constraint, so

$$\begin{aligned} p_x \cdot x^* + p_y \cdot y^* &= w \\ 2 \cdot x^* + 3 \cdot y^* &= 30 \\ y^* &= 10 - \frac{2}{3}x^*. \end{aligned}$$

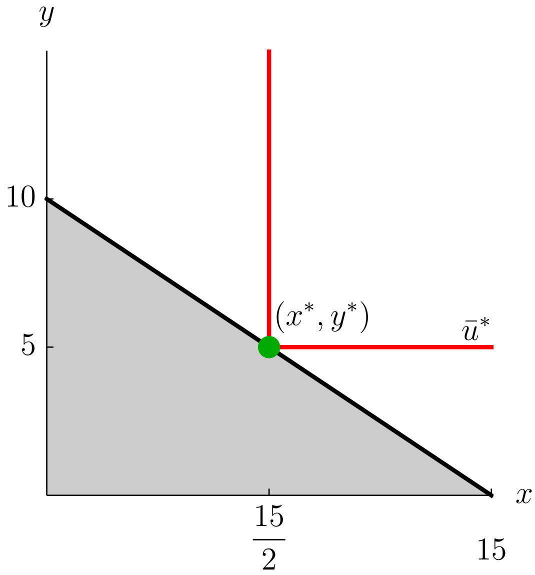
Second, we consume at the optimal ratio, so

$$2x^* = 3y^*.$$

Finally, solving the two simultaneously gives:

$$(x^*, y^*) = (7.5, 5).$$

An example with perfect complements (3)

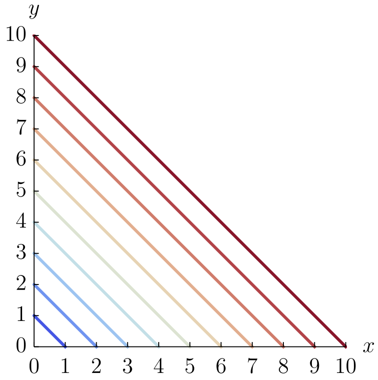


Where are we now?

The problem with perfect substitutes

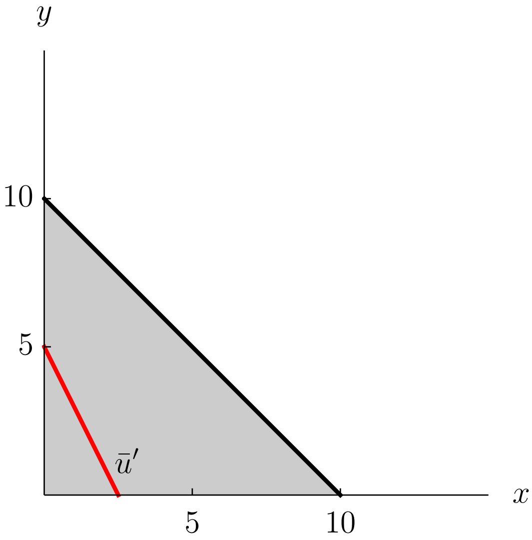
As always, the first condition of **the golden rule**, that the optimum is on the budget constraint, still works. But we cannot equate slopes:

- Why? Recall the indifference curves . . .

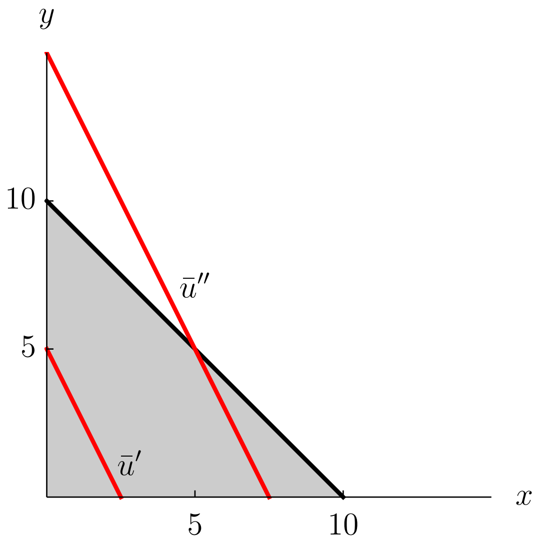


- The slope is constant.
- Either never equals p_x/p_y or always does.

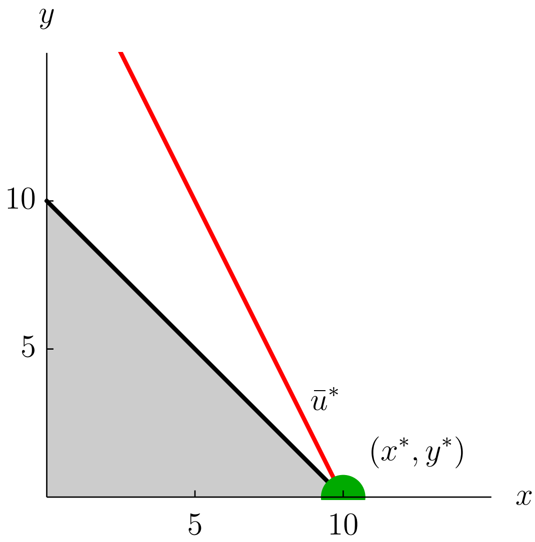
Scenario 1: $\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$ (1)



Scenario 1: $\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$ (2)



Scenario 1: $\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$ (3)



Conclusions for Scenarios 1 and 2

Conclusion for Scenario 1:

$$\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$$

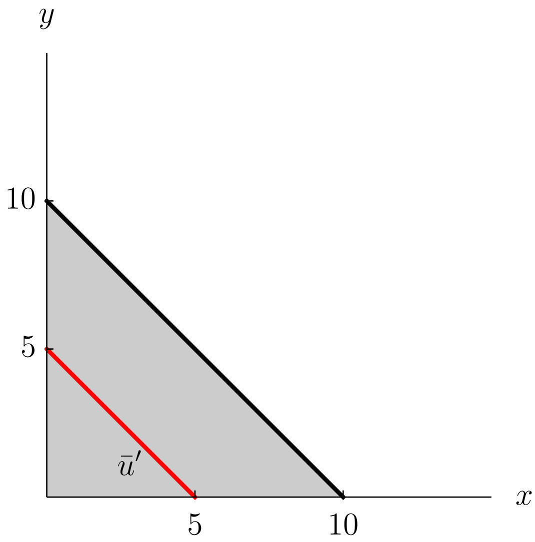
x is better value than y so spend all of your money on x.
Therefore, optimal bundle is $\left(\frac{w}{p_x}, 0\right)$

Conclusion for Scenario 2: (by same logic)

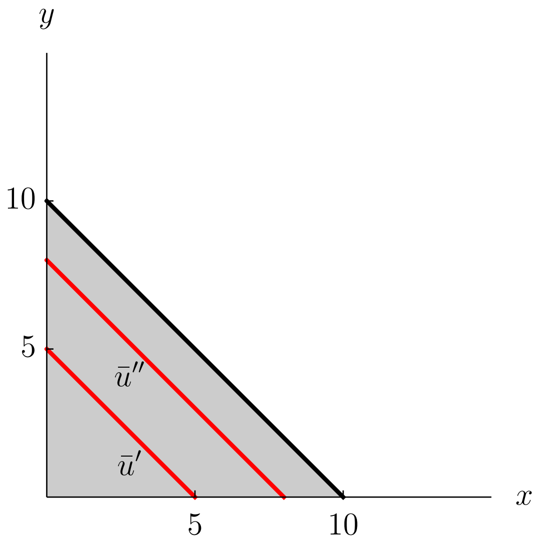
$$\frac{MU_x}{p_x} < \frac{MU_y}{p_y}$$

y is better value than x so spend all of your money on y.
Therefore, optimal bundle is $\left(0, \frac{w}{p_y}\right)$

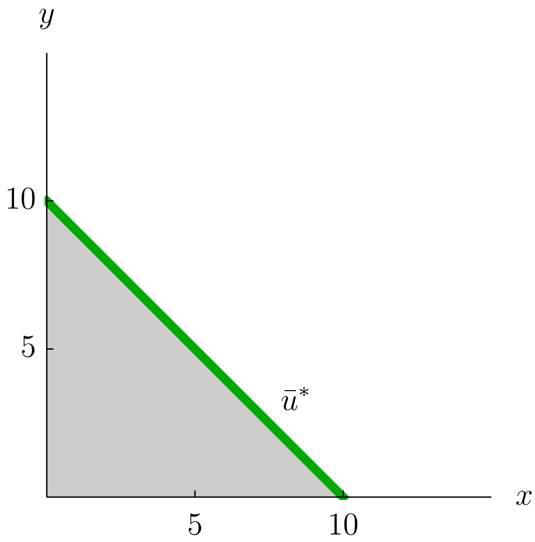
Scenario 3: $\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$ (1)



Scenario 3: $\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$ (2)



Scenario 3: $\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$ (3)



Conclusions for Scenario 3

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

x and y are equally good value, so any combination of x and y whose total cost is w is optimal.

- So $\left(\frac{w}{p_x}, 0\right)$ is optimal.
- As is $\left(0, \frac{w}{p_y}\right)$.
- And anything in between . . .

Where are we now?

A generalized golden rule

(x^*, y^*) is the solution to two equations:

1. Equation 1 (BC): $p_x \cdot x^* + p_y \cdot y^* = w^*$.
2. Equation 2:
   - For Cobb-Douglas and logarithmic: $MRS_{xy}(x^*, y^*) = \frac{p_x}{p_y}$.
   - For perfect complements $(u = \min(a \cdot x, b \cdot y))$: $a \cdot x = b \cdot y$.
   - For perfect substitutes: consume only the good that has the highest *bang per buck*., i.e.,
     - if $\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$, $x^* = \frac{w}{p_x}$.
     - if $\frac{MU_x}{p_x} < \frac{MU_y}{p_y}$, $y^* = \frac{w}{p_y}$.
     - if $\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$, no additional equation.

Where are we now?

When prices change . . .

Suppose we decrease the price of one good, leaving the other constant. The optimal bundle changes for two reasons:

Income effect

The increase in consumption of either or both goods because real wealth is now greater, i.e., we can afford more.

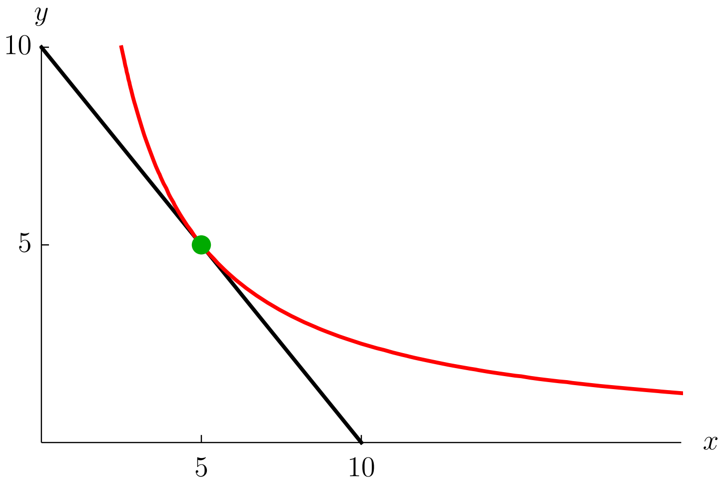
Substitution effect

The increase in consumption of the good whose price was reduced (and decrease in consumption of the other good) because the relative prices of the goods has changed.

We can calculate the two separately.

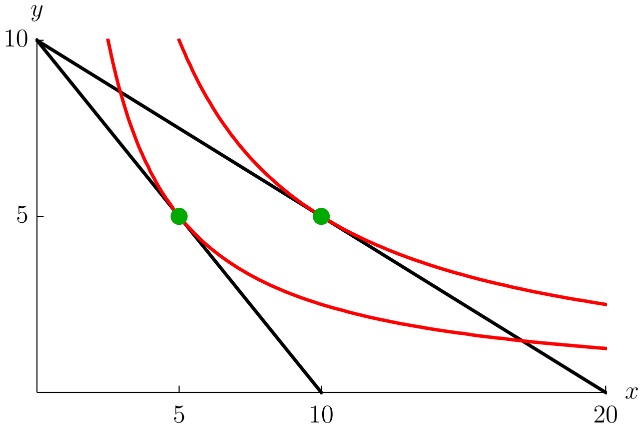
An example of IE & SE (1)

Suppose $p_x = 2$, $p_y = 2$, $w = 20$, and $u(x, y) = \ln x + \ln y$:



An example of IE & SE (2)

Now suppose p_x drops from 2 to 1:



Total change in quantity demanded of x is 5, of y is 0.

An example of IE & SE (3)

To separate the two effects, we need to create a hypothetical:

- How much of each good would we consume if we were required to remain on the original indifference curve but faced the new price ratio?

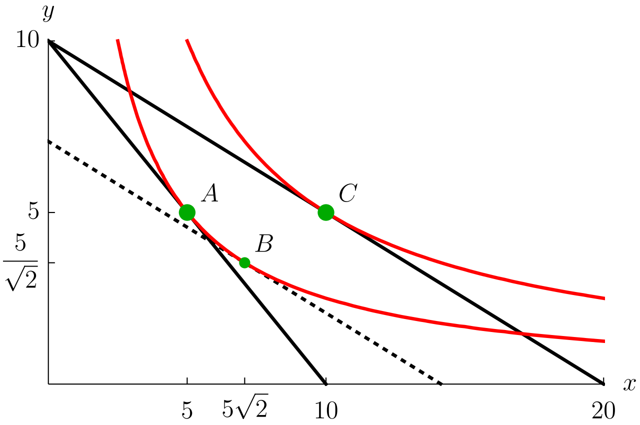
Let:

- A denote original (before price change) optimal bundle.
- B denote the hypothetical bundle.
- C denote final (after price change) optimal bundle.

The substitution effect is the difference between A and B. The income effect is the difference between B and C.

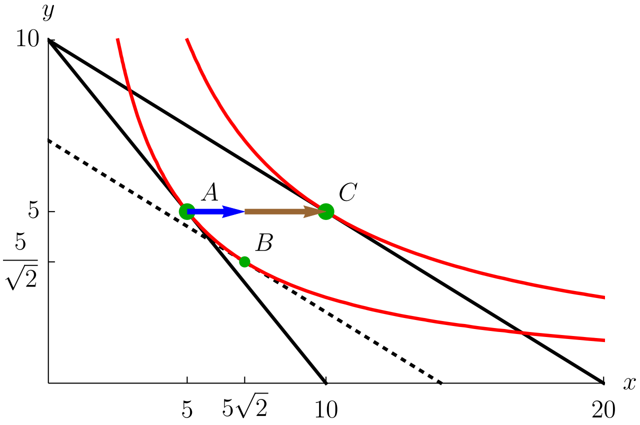
An example of IE & SE (4)

Now representing all three bundles:



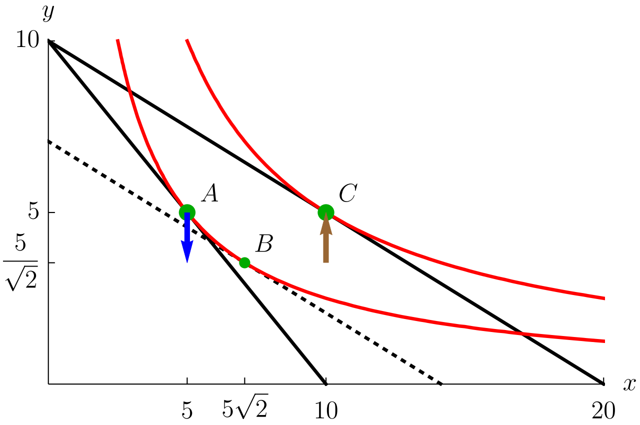
An example of IE & SE (5)

Substitution and income effects for x:



An example of IE & SE (6)

Substitution and income effects for y:



An example of IE & SE (7)

Table of substitution and income effects for x and y resulting from the change from $p_x = 2$ to $p_x = 1$ (holding $p_y = 2$ constant):

	x	y
Income effect:	$10 - 5\sqrt{2}$	$5 - \frac{5}{\sqrt{2}}$
Substitution effect:	$5\sqrt{2} - 5$	$\frac{5}{\sqrt{2}} - 5$
Total effect:	5	0