NU Econ 101: Lecture 5

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Where are we now?

Demand curve

For any prices and preferences, we can find an agent's optimal bundle, i.e., how much of each good they demand.

Now, we want to work out how much of a given good an agent would demand for a variety of different prices.

- We will call this a **demand curve**.
  - A demand curve for x is in (x, p_x)-space

We can do this in a couple of different ways:

1. Work out demand for a few prices and connect dots.
2. Solve explicitly for a demand function.

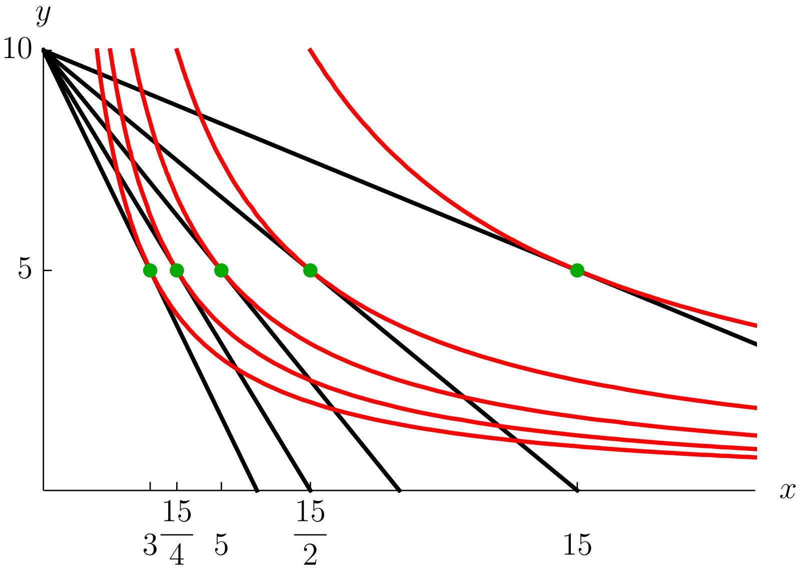
Our working example

To have an example to work with, let's assume the following:

- Logarithmic utility: $u(x, y) = \ln x + \ln y$
- $p_y = 3$
- $w = 30$

We want to create a demand curve for x for prices of x varying between $p_x = 1$ and $p_x = 5$.

Method 1: Find a few points and interpolate (1)

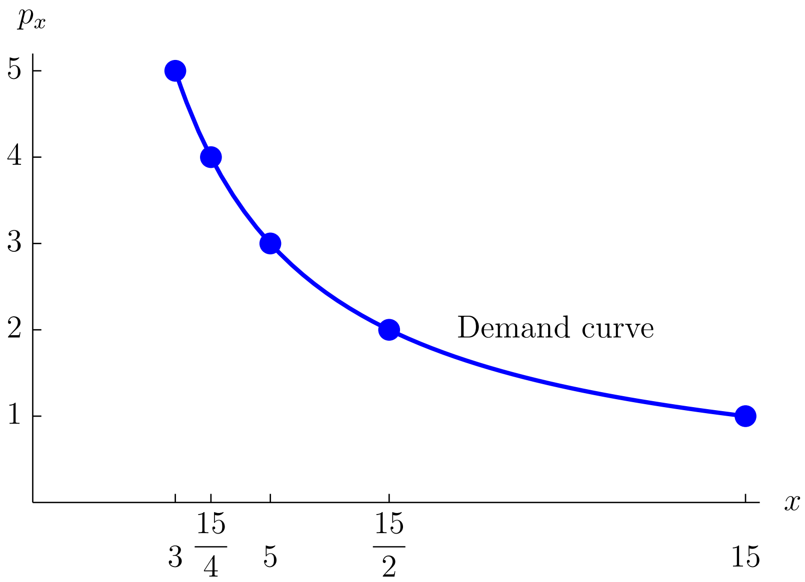


Method 1: Find a few points and interpolate (2)

We want to compile those points into a table to use to create a demand curve:

p_x	x^*
1	15
2	7.5
3	5
4	3.75
5	3

Method 1: Find a few points and interpolate (3)



Method 2: Solve explicitly for $x^*(p_x)$. (1)

Solving the problem a bunch of times is no fun. Instead, let's solve for x^* as a function of p_x so that we know the demand for x for any p_x.

We can still use our two golden rule equations.

- We just do not enter in a number for p_x.

Method 2: Solve explicitly for $x^*(p_x)$. (2)

Equation 1 (BC):

$$p_x \cdot x^* + p_y \cdot y^* = w$$
$$p_x \cdot x^* + 3 \cdot y^* = 30$$
$$y^* = 10 - \frac{p_x}{3} \cdot x^*$$

Equation 2 (Equating slopes / bang per buck):

$$\frac{MU_x(x^*, y^*)}{p_x} = \frac{MU_y(x^*, y^*)}{p_y}$$
$$\frac{1/x^*}{p_x} = \frac{1/y^*}{3}$$
$$y^* = \frac{p_x}{3} \cdot x^*$$

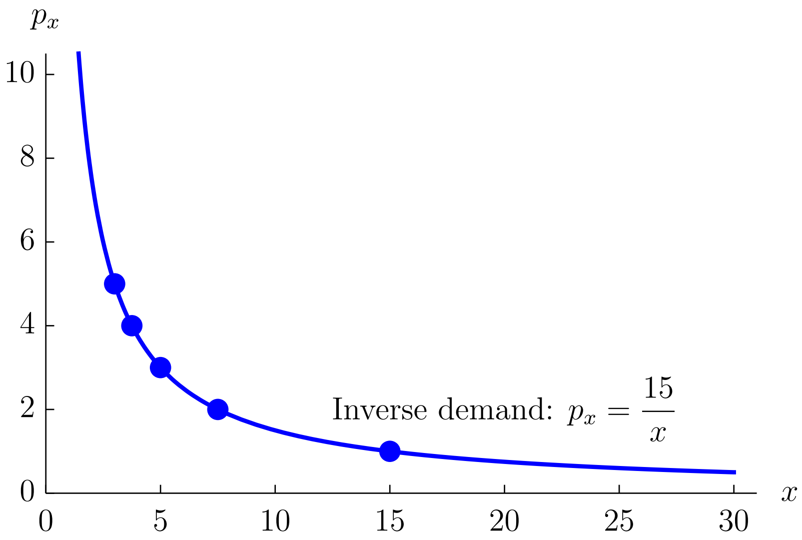
Method 2: Solve explicitly for $x^*(p_x)$. (3)

Since we don't care about y^*, just take the right-hand side of each of the two equations above and equate them:

$$\begin{aligned} 10 - \frac{p_x}{3} \cdot x^* &= \frac{p_x}{3} \cdot x^* \\ \frac{2}{3} \cdot p_x \cdot x^* &= 10 \\ x^* &= \frac{3}{2} \cdot \frac{10}{p_x} \\ x^* &= \frac{15}{p_x} \end{aligned}$$

Now we know x^* for any p_x and we can plot it ...

Method 2: Solve explicitly for $x^*(p_x)$. (4)



A few notes on demand

1. If $x = 15/p_x$ is the **demand function**. Then $p_x = 15/x$ is the **inverse demand function**.
   - Since we're plotting in (x, p_x)-space, easier to use inverse demand.
2. Did you notice that $y^* = 5$ for all p_x we tried?
   - This is a neat feature of logarithmic utility: your consumption of one good is independent of the price of the other.
3. Are we still limiting ourselves to two goods?
   - Formally, yes. Informally, not really. You can think of y as being a composite *all goods other than* x.
     - Therefore, this is somewhat equivalent to calculating a demand function for x in a world with many goods.

Where are we now?

Market demand

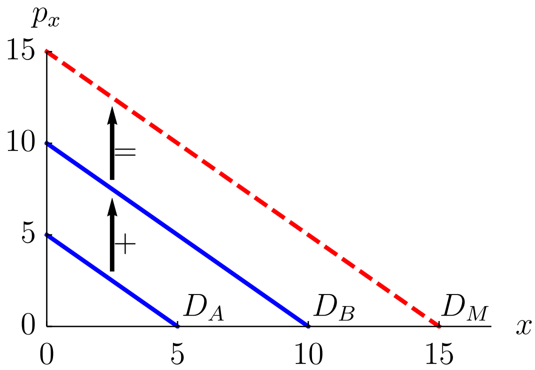
Once we have individual demands, we can sum across them to get market demand.

Suppose we have two households, A and B. Let:

- D_A denote household A's demand function.
  - For household A, inverse demand: $p_x = 5 - x_A^*$
- D_B denote household B's demand function.
  - For household B, inverse demand: $p_x = 10 - x_B^*$
- $D_M = D_A + D_B$ denote the market demand function.

WRONG: Summing vertically

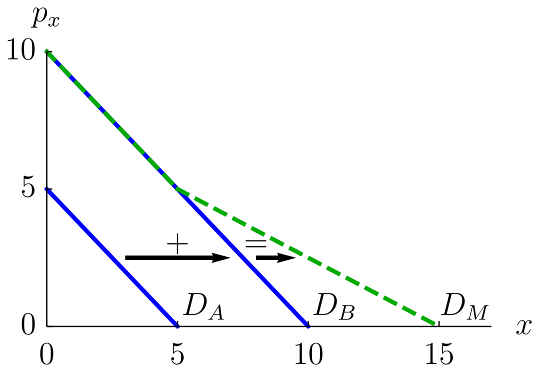
Do not sum vertically!



That is summing inverse demands instead of demands.

CORRECT: Summing horizontally

Sum horizontally!



That is summing the actual demands.

Summing demand functions (1)

In the last few slides, we saw how to aggregate to get a market demand graphically.

Mathematically, we just need to make sure we sum demand functions, not inverse demand functions.

Summing demand functions (2)

So, take the two inverse demand function and write them as demand functions:

- For A, $p_x = 5 - x_A^* \implies x_A^* = 5 - p_x$
- For B, $p_x = 10 - x_B^* \implies x_B^* = 10 - p_x$
- Then $x_M^* = x_A^* + x_B^* = 15 - 2p_x$.

Finally reverse it back to an inverse market demand for plotting:

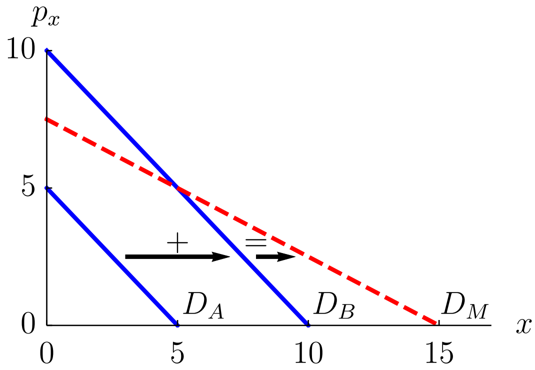
$$p_x = 7.5 - \frac{x_M^*}{2}$$

BUT THIS IS WRONG!

- Can anybody work out why? (Let's look at it . . .)

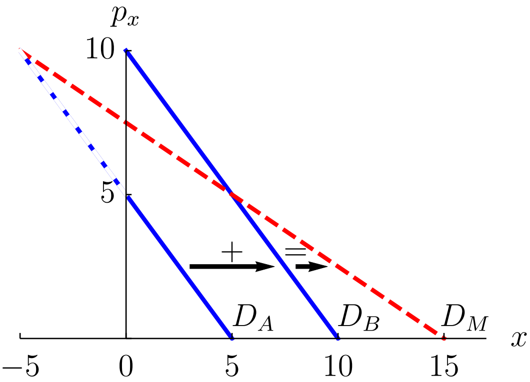
Summing demand functions (3)

What went wrong?



Summing demand functions (4)

The problem: $x_A^* = 5 - p_x$ suggests negative demand at $p_x > 5$



That is how we get the incorrect market demand.

Summing demand functions (5)

Technically we should actually add these two:

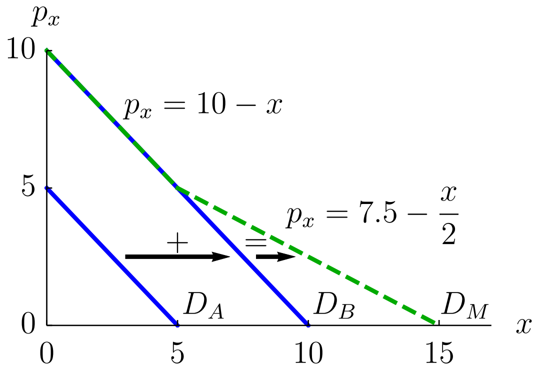
- $x_A^* = \max(5 - p_x, 0)$
- $x_B^* = \max(10 - p_x, 0)$

But it's probably easier to just note that:

- For $p_x > 5$, $x_M^* = x_B^*$ (only B participates in market).
  - So we plot $p_x = 10 - x$ for $x < 5$.
- For $p_x < 5$, $x_M^* = x_A^* + x_B^*$ (both participate in market).
  - So we plot $p_x = 7.5 - \frac{x}{2}$ for $x \geq 5$.

Summing demand functions (6)

That gets us back to the right answer:



Where are we now?

Categorizing goods (1)

Let's define a couple of types of goods (with regards to responsiveness to **income** changes):

Normal goods

Goods for which demand goes **up** when income/wealth goes up.

- Examples: Most goods...

Inferior goods

Goods for which demand goes **down** when income/wealth goes up.

- Examples: Potatoes, bus rides, canned products

Categorizing goods (2)

Let's define a couple of types of goods (with regards to responsiveness to **price** changes):

Ordinary goods

Goods for which demand goes **down** when the gd's price goes up.

- Examples: *Probably* all goods.

Giffen goods

Goods for which demand goes **up** when that good's price goes up.

- Arguments exist for: Potatoes, kerosene, *shochu*.
    - Arguments certainly supportable on an individual level, less so on aggregate.

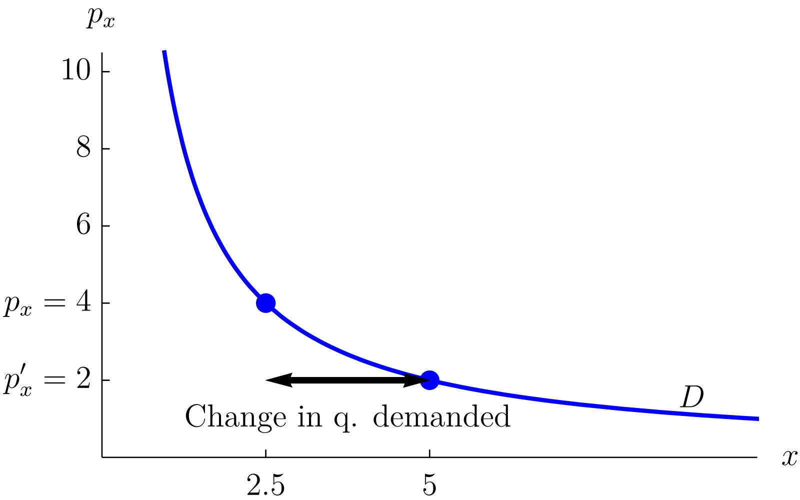
Changes in quantity demanded (1)

If the price of a good changes, this **doesn't** change it's demand curve.

- Remember the demand curve gives demand for *all* prices.

We call the result a **change in quantity demanded** as it is a movement along the demand curve.

Changes in quantity demanded (2)



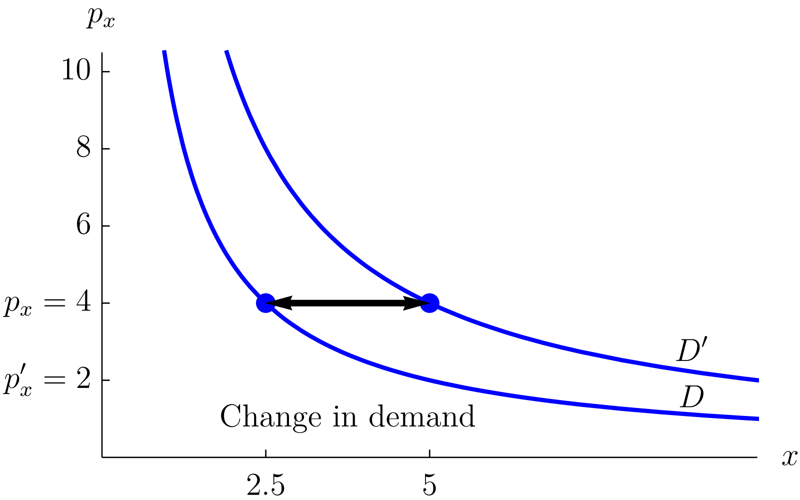
Changes in demand (1)

If the prices of other goods change or income/wealth changes, this **does** change a good's demand curve.

- Remember the demand curve is for a specific income/wealth level and prices of other goods.

We call the result a **change in demand** as it generates a new demand curve.

Changes in demand (2)



Consequences of changes (1)

Suppose x is an inferior, ordinary good.

What will happen to x^* if p_x goes up?

- a) Increase in quantity demanded of x.
- b) Decrease in quantity demanded of x.
- c) Increase in demand of x.
- d) Decrease in demand of x.

Solution: **b)** because increasing p_x doesn't change demand function, so change is in quantity demanded, and it is negative because x is ordinary.

- Inferior is irrelevant to the question.

Consequences of changes (2)

Suppose x and y are ordinary complements.

What will happen to y^* if p_x goes down?

- a) Increase in quantity demanded of y.
- b) Decrease in quantity demanded of y.
- c) Increase in demand of y.
- d) Decrease in demand of y.

Solution: **c)** because p_x decreasing increases the quantity demanded of x (because it is ordinary) which increases demand for y, its complement. It is c) not a) because the demand function of y does not depend on p_x, just p_y.

Consequences of changes (3)

Suppose x is inferior and y is normal.

What will happen to x^* and y^* as w increases?

- a) Demand for both increases.
- b) Demand for both decreases.
- c) Demand for x increases, y decreases.
- d) Demand for x decreases, y increases.

Solution: **d)** basically by definition. Note that these are changes to demand as w is changing, not a price.

Where are we now?

Price elasticity of demand

Remember how we wanted to know how utility changed as quantity changed.

- It turned out this was just *marginal utility*.
    - Marginal utility was critical in analyzing choice.

We might also want to know how demand changes as prices do.

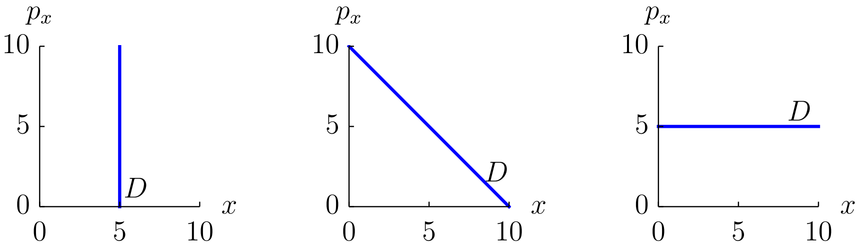
- If we just analyze the slope, units matter.
    - Instead we compare percentage changes.

Price elasticity of demand

$$\text{price elasticity of demand} = -\frac{\%\ \text{change in quantity demanded}}{\%\ \text{change in price}}$$

Describing elasticity

These are a few key elasticities, with their names and (values):



Perfectly inelastic (0) | Unit elastic (1) | Perfectly elastic (∞)

A good is said to be be:

- *elastic* if elasticity is greater than 1.
- *inelastic* if elasticity is less than 1.

Where are we now?

Choosing labor supply (1)

Individuals don't just consume, they also work to earn money to consume.

- A key decision an individual might make is how much to work.

An example:

Suppose we gain utility from consuming goods (c) and hours of relaxing (r):

$$u(c, r) = c \cdot r$$

Suppose the hourly wage we get is $w = 4$ and the $p_c = 1$.

- We'll say you work h hours (you choose h).

Choosing labor supply (2)

We know we want to maximize utility, but what are the constraints?

- Budget constraint: $p_c \cdot c \leq w \cdot h$
- Time constraint: $h + r \leq 24$

Optimally, we will spend all of our money and use all of our time.

Solving for optimal labor supply:

Step 1: Combine constraints by rewriting $h = 24 - r$ and subbing that in to the budget constraint to get $p_c \cdot c = w(24 - r)$.

Choosing labor supply (3)

Step 2: Plug in $p_c = 1$ to get $c = 4(24 - r)$ and substitute that in for c in the utility function to get it as a function of just r

$$u(r) = 4(24 - r) \cdot r = 96r - 4r^2$$

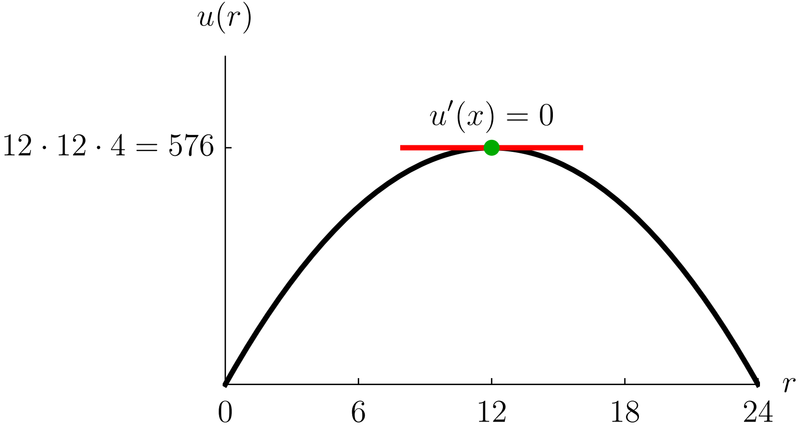
Step 3: Find r^* by differentiating $u(r)$ and setting the derivative equal to zero and solving (this is a first order condition).

- (Don't worry – I won't make you do this.)

$$u'(r) = 96 - 8r = 0 \quad \Longrightarrow \quad r^* = 12$$

Plugging this back into constraints gives $h^* = 12$ and $c^* = 48$.

Choosing labor supply (4)



Where are we now?

End of Part 1: Consumer theory

We have now finished **Part 1** of the course, on consumers.

Tomorrow we start **Part 2**, on producers.