ECON 101, Problem Set 2
Due Tuesday, June 14

1. Suppose there are two divisible goods, x and y, and your utility function is $u(x, y) = \ln(x) + 4\ln(y)$, $p_x = 4$, and $w = 40$. Note the following derivatives:

$$\frac{d}{dx}(\ln(x) + 4\ln(y)) = \frac{1}{x} \quad \text{and} \quad \frac{d}{dy}(\ln(x) + 4\ln(y)) = \frac{4}{y}$$

   (a) Supposing $p_y = 2$, and looking at the prices and utility function, do you think you would want to consume more x or more y? In a sentence or two, give an answer using only your intuition, i.e., without solving.

   (b) Now, solve for the optimal consumption bundle.

   (c) Finally, find the formulas for the demand curve for y and the inverse demand curve for y. You do not need to plot them.

2. Suppose there are two divisible goods, x and y, and your utility function is $u(x, y) = 2x+7y$, $p_x = 3$, $p_y = 10$, and $w = 30$.

   (a) Draw two indifference curves (one with utility 14 and the other with utility 28) for these preferences. Add the budget constraint to this same plot. Make sure to label the axes and all intercepts (of each of the three lines with each of the axes).

   (b) If one exists, give an example of a specific bundle of x and y that is worth exactly 14 utils and affordable. If no such bundle exists, explain using the plot you have drawn.

   (c) If one exists, give an example of a specific bundle of x and y that is worth exactly 28 utils and affordable. If no such bundle exists, explain using the plot you have drawn.

   (d) Using the generalized golden rule of optimal consumption, find the optimal bundle, (x^*, y^*). Note the following:

$$\frac{d}{dx}(2x + 7y) = 2 \quad \text{and} \quad \frac{d}{dy}(2x + 7y) = 7$$

   (e) (a challenge!) Now suppose we want to know demand of x for all p_x. Keep assuming the same utility fn., the same p_y and the same w. Plot the inverse demand curve for x (make sure you use the right axes) for p_x between 1 and 4 and x between 0 and 15.

3. Suppose there are two goods, x and y, for which ALL of the following facts are true:

   (a) When p_x goes up, the demand for x increases.

   (b) When w increases, the demand for x decreases.

   (c) When p_y goes down, the quantity of y demanded increases.

   (d) When p_x goes up, the quantity of y demanded decreases.

   (e) Utility is increasing in each of the goods – i.e. more is better – and there are no other goods besides x and y.

   First, note that two of the "facts" above have confused the difference between changes in quantity demanded and changes in demand. Identify which two "facts" use the wrong terminology and correct them. Now, for each of the following statements, decide if it is true, false, or that we do not have enough information above to determine. Briefly (in a sentence or two) explain each answer.

   (a) x and y are complements (a challenge).

   (b) y is an ordinary good.

   (c) x is a Giffen good.

   (d) x is an inferior good.

   (e) y is an inferior good (a challenge).

   (f) When w and p_x both increase, demand for x increases.

4. Suppose Nelson Inc. has Cobb-Douglas production function $F(K, L) = K^{\frac{1}{2}} \cdot L^{\frac{1}{2}}$, $p_K = 2$, and $p_L = 3$. Note that:

$$\frac{d}{dK}(K^{\frac{1}{2}} \cdot L^{\frac{1}{2}}) = \frac{L^{\frac{1}{2}}}{2K^{\frac{1}{2}}} \quad \text{and} \quad \frac{d}{dL}(K^{\frac{1}{2}} \cdot L^{\frac{1}{2}}) = \frac{K^{\frac{1}{2}}}{2L^{\frac{1}{2}}}$$

   (a) Use the α-technique we did in the lectures to find whether Nelson Inc.'s production function exhibits increasing, decreasing, or constant returns to scales.

   (b) Find the formula for Nelson's cost curve. It will be a little ugly, so do not plot it.