ECON 101, Problem Set 2
Due Tuesday, June 14

1. Suppose there are two divisible goods, x and y, and your utility function is $u(x, y) = \ln(x) + 4\ln(y)$, $p_x = 4$, and $w = 40$. Note the following derivatives:

$$\frac{d}{dx}(\ln(x) + 4\ln(y)) = \frac{1}{x} \quad \text{and} \quad \frac{d}{dy}(\ln(x) + 4\ln(y)) = \frac{4}{y}$$

(a) Supposing $p_y = 2$, and looking at the prices and utility function, do you think you would want to consume more x or more y? In a sentence or two, give an answer using only your intuition, i.e., without solving.

Solution:

The price of y is lower than the price of x and there is a coefficient of 4 on the $\ln(y)$ term which is bigger than the coefficient of 1 on the $\ln(x)$ term, so y is cheaper and seems to provide a bit more utility, so we would expect that $y^* > x^*$. On the other hand, we know we will consume some x, otherwise our utility will be infinitely negative!

(b) Now, solve for the optimal consumption bundle.

Solution:

The first equation we use is the budget constraint:

$$\begin{aligned} p_x \cdot x^* + p_y \cdot y^* &= w \\ 4 \cdot x^* + 2 \cdot y^* &= 40 \\ y^* &= 20 - 2x^* \end{aligned}$$

The second equation we use is equating the bang per buck (or, equivalently, equating the slopes):

$$\begin{aligned} \frac{MU_x(x^*, y^*)}{p_x} &= \frac{MU_y(x^*, y^*)}{p_y} \\ \frac{1/x^*}{4} &= \frac{4/y^*}{2} \\ 2/x^* &= 16/y^* \\ y^* &= 8x^* \end{aligned}$$

Equating the RHS of the two equations:

$$\begin{aligned} 20 - 2x^* &= 8x^* \\ 20 &= 10x^* \\ x^* &= 2 \end{aligned}$$

Plugging that into $y^* = 8x^*$ gives us that $y^* = 16$ so the optimal consumption bundle is $(x^*, y^*) = (2, 16)$.

(c) Finally, find the formulas for the demand curve for y and the inverse demand curve for y. You do not need to plot them.

Solution:

The first equation we use is the budget constraint:

$$\begin{aligned} p_x \cdot x^* + p_y \cdot y^* &= w \\ 4 \cdot x^* + p_y \cdot y^* &= 40 \\ x^* &= 10 - \frac{p_y}{4} \cdot y^* \end{aligned}$$

The second equation we use is equating the bang per buck (or, equivalently, equating the slopes):

$$\begin{aligned} \frac{MU_x(x^*, y^*)}{p_x} &= \frac{MU_y(x^*, y^*)}{p_y} \\ \frac{1/x^*}{4} &= \frac{4/y^*}{p_y} \\ p_y/x^* &= 16/y^* \\ x^* &= \frac{p_y}{16} \cdot y^* \end{aligned}$$

Equating the RHS of the two equations:

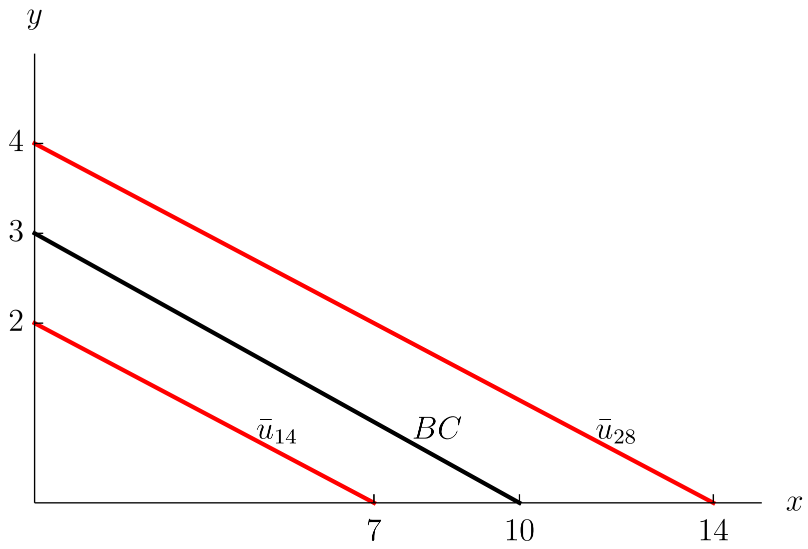
$$\begin{aligned} 10 - \frac{p_y}{4} \cdot y^* &= \frac{p_y}{16} \cdot y^* \\ y^* \cdot \left(\frac{p_y}{4} + \frac{p_y}{16}\right) &= 10 \\ y^* \cdot \left(\frac{5p_y}{16}\right) &= 10 \\ y^* &= \frac{32}{p_y} \end{aligned}$$

That's the demand function and the inverse demand function is just $p_y = \frac{32}{y}$.

2. Suppose there are two divisible goods, x and y, and your utility function is $u(x, y) = 2x+7y$, $p_x = 3$, $p_y = 10$, and $w = 30$.

   (a) Draw two indifference curves (one with utility 14 and the other with utility 28) for these preferences. Add the budget constraint to this same plot. Make sure to label the axes and all intercepts (of each of the three lines with each of the axes).

   Solution:



   (b) If one exists, give an example of a specific bundle of x and y that is worth exactly 14 utils and affordable. If no such bundle exists, explain using the plot you have drawn.

   Solution:

   That indifference curve is below the budget constraint, so all points on it are affordable. One such point is $(x, y) = (7, 0)$, but you could give any point on the line as an answer here.

   (c) If one exists, give an example of a specific bundle of x and y that is worth exactly 28 utils and affordable. If no such bundle exists, explain using the plot you have drawn.

   Solution:

   The entire indifference curve for 28 utils is outside of the budget set (i.e. above and to the right of the BC), therefore no point on it is affordable.

   (d) Using the generalized golden rule of optimal consumption, find the optimal bundle, (x^*, y^*). Note the following:

$$\frac{d}{dx}(2x + 7y) = 2 \quad \text{and} \quad \frac{d}{dy}(2x + 7y) = 7$$

   Solution:

   To know which good is better value, we compare the *bangs per bucks*,

$$\frac{MU_x}{p_x} = \frac{2}{3} < \frac{7}{10} = \frac{MU_y}{p_y},$$

   and find that y is better value. Therefore, the optimal bundle is

$$(x^*, y^*) = \left(0, \frac{w}{p_y}\right) = (0, 3).$$

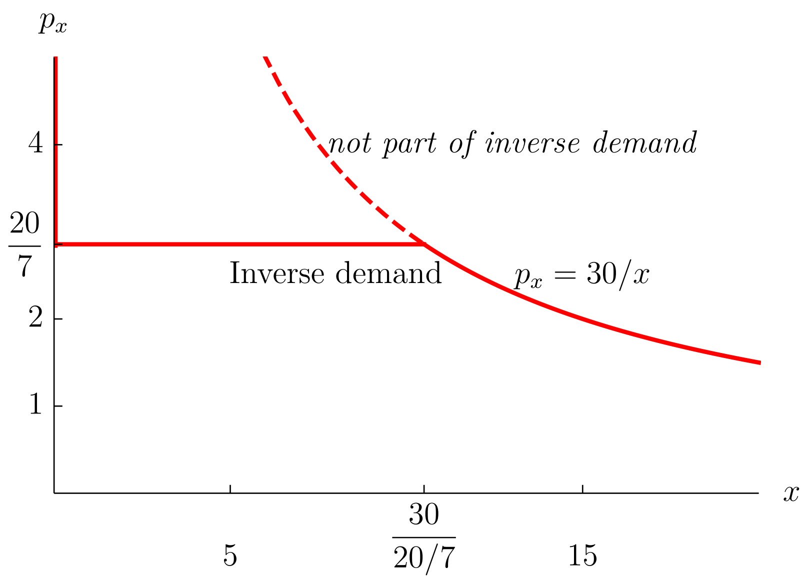
(e) (a challenge!) Now suppose we want to know demand of x for all p_x. Keep assuming the same utility fn., the same p_y and the same w. Plot the inverse demand curve for x (make sure you use the right axes) for p_x between 1 and 4 and x between 0 and 15.

Solution:

This is actually a little trickier than it seems as the demand curve is discontinuous. Remember, with perfect substitutes, if we get better *bang per buck* from y than x, then we consume no x at all. But as soon as p_x is low enough that x is the better value, we consume only x. The spot where it flips is right where their values are equal. So let's solve for that cutoff value of p_x:

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$
$$\frac{2}{p_x} = \frac{7}{10}$$
$$p_x = 20/7$$

So, with $p_x > 20/7$, $x^* = 0$. For $p_x < 20/7$, we spend all of our money on x, so $x^* = w/p_x$. Remember, we plot the inverse demand curve, so that is $p_x = w/x = 30/x$. Putting this all together in a plot gives us:



3. Suppose there are two goods, x and y, for which ALL of the following facts are true:

   (a) When p_x goes up, the demand for x increases.

   (b) When w increases, the demand for x decreases.

   (c) When p_y goes down, the quantity of y demanded increases.

   (d) When p_x goes up, the quantity of y demanded decreases.

   (e) Utility is increasing in each of the goods – i.e. more is better – and there are no other goods besides x and y.

   First, note that two of the "facts" above have confused the difference between changes in quantity demanded and changes in demand. Identify which two "facts" use the wrong terminology and correct them.

   Solution:

   Fact (a) uses the wrong terminology. Since p_x is changing and affecting x, that should be a change in quantity demanded (a movement along the demand curve), not demand. Fact (d) also uses the wrong terminology. Since the price of the other good is changing, that should be a change in demand, not quantity demanded.

   Now, for each of the following statements, decide if it is true, false, or that we do not have enough information above to determine. Briefly (in a sentence or two) explain each answer.

   (a) x and y are complements (a challenge).

   Solution:

   This is false. They are actually substitutes. When p_x goes up, the quantity of x demanded increases by fact (a). Combine that with fact (d), which says that when p_x goes up, demand for y decreases. So, we have a situation in which the quantity demanded of x is increasing as the demand for y is decreasing, hence they are substitutes.

   (b) y is an ordinary good.

   Solution:

   This is true by fact (c).

   (c) x is a Giffen good.

   Solution:

   This is true by fact (a).

   (d) x is an inferior good.

   Solution:

   This is true by fact (b)

   (e) y is an inferior good (a challenge).

   Solution:

   This is false as we can conclude y is normal. It seems like we do not have enough information to tell, as none of the facts specifies directly what happens to demand for y when w increases. However, using fact (e) and the fact that x is inferior, we can actually conclude that y cannot possibly be inferior. If both goods were inferior, then, as income went up, demand for both would decrease, but then you wouldn't be spending all of your money, which contradicts face (e). The lesson here is that it's impossible for all of the goods you consume to be inferior. At least one must be normal.

(f) When w and p_x both increase, demand for x increases.

Solution:

We do not have enough information to answer this. When w goes up, demand for x decreases. When p_x goes up, the quantity of x demanded decreases. So the sign of the total effect depends how much each is going up. Note this double change is a change in demand because w is changing. The change in p_x yields a change in quantity demanded, but since there's also a change in w, it is still a change in the demand curve itself also.

4. Suppose Nelson Inc. has Cobb-Douglas production function $F(K, L) = K^{\frac{1}{2}} \cdot L^{\frac{1}{2}}$, $p_K = 2$, and $p_L = 3$. Note that:

$$\frac{d}{dK}(K^{\frac{1}{2}} \cdot L^{\frac{1}{2}}) = \frac{L^{\frac{1}{2}}}{2K^{\frac{1}{2}}} \quad \text{and} \quad \frac{d}{dL}(K^{\frac{1}{2}} \cdot L^{\frac{1}{2}}) = \frac{K^{\frac{1}{2}}}{2L^{\frac{1}{2}}}$$

(a) Use the α-technique we did in the lectures to find whether Nelson Inc.'s production function exhibits increasing, decreasing, or constant returns to scales.

Solution:

$$\begin{aligned} F(\alpha \cdot K, \alpha \cdot L) &= (\alpha \cdot K)^{\frac{1}{2}} \cdot (\alpha \cdot L)^{\frac{1}{2}} \\ &= \alpha \cdot K^{\frac{1}{2}} \cdot L^{\frac{1}{2}} \\ &= \alpha \cdot F(K, L) \end{aligned}$$

Hence Nelson has constant returns to scale.

(b) Find the formula for Nelson's cost curve. It will be a little ugly, so do not plot it.

Solution:

I'm just going to omit the stars on the variables throughout to make it a little easier to read, but these are all optimal values that we're solving for.

The first equation we use is the isoquant:

$$\begin{aligned} F(K, L) &= Y \\ K^{\frac{1}{2}} \cdot L^{\frac{1}{2}} &= Y \end{aligned}$$

The second equation we use is that we equate *bang per buck*, or, equivalently, the slopes of the isoquant and isocost.

$$\begin{aligned} \frac{MP_K(K^*, L^*)}{p_K} &= \frac{MP_L(K^*, L^*)}{p_L} \\ \frac{L^{\frac{1}{2}}}{2K^{\frac{1}{2}} \cdot p_K} &= \frac{K^{\frac{1}{2}}}{2L^{\frac{1}{2}} \cdot p_L} \\ L \cdot p_L &= K \cdot p_K \\ L &= \frac{2}{3} \cdot K \end{aligned}$$

Combining these two gives us:

$$K^{\frac{1}{2}} \cdot \left(\frac{2}{3} \cdot K\right)^{\frac{1}{2}} = Y$$
$$K^* = \frac{Y}{\left(\frac{2}{3}\right)^{\frac{1}{2}}}$$

And $L^* = \frac{2}{3}K^*$ so $L^* = \frac{2}{3} \cdot \frac{Y}{\left(\frac{2}{3}\right)^{\frac{1}{2}}}$. And the cost function is:

$$C(Y) = p_K \cdot K^* + p_L \cdot L^* = 2 \cdot \frac{Y}{\left(\frac{2}{3}\right)^{\frac{1}{2}}} + 2 \cdot \frac{Y}{\left(\frac{2}{3}\right)^{\frac{1}{2}}} = \frac{4Y}{\sqrt{2/3}} = \frac{\sqrt{3} \cdot 4 \cdot Y}{\sqrt{2}} = 2\sqrt{6} \cdot Y$$

It's fine if you didn't simplify ...