ECON 101, Problem Set 3
Due Tuesday, June 28

1. Suppose market supply is $q_S = 2p - 100$ and market demand is $q_D = 200 - 2p$.
   (a) Find market-clearing price and quantity. Plot the curves and label producer and consumer surplus.
   (b) Calculate producer and consumer surplus.
   (c) Now suppose the government mandates a price floor of 80. Draw a new plot to reflect these changes, showing the new surpluses and deadweight loss.
   (d) Calculate the new surpluses and deadweight loss.

2. Nelson runs a company that makes chips (x) and pickles (y). He has four employees:
   - Anne can make 5 chips or 3 pickles per hour.
   - Bob can make 4 chips or 2 pickles per hour.
   - Cathy can make 3 chips or 5 pickles per hour.
   - Derek can make 2 chips or 4 pickles per hour.

   Each of the employees works a ten-hour day.
   (a) Who has the absolute advantage in each of the goods?
   (b) Rank the workers based upon their comparative advantage in pickles.
   (c) If all four workers work on pickles, how many pickles will they make in one day.
   (d) If three workers work on pickles, which three should it be. What is the resulting output of chips and pickles?
   (e) If two workers work on pickles, which two should it be. What is the resulting output of chips and pickles?
   (f) If one worker works on pickles, which one should it be. What is the resulting output of chips and pickles?
   (g) If no worker works on pickles, how many chips will they make in one day?
   (h) Plot the five points you just found in (x, y) space. Connect the dots with straight-lines. If you have done it correctly, this is Nelson's PPF. Observe that using the workers according to their comparative advantage allows us to produce more output than if we had used them interchangeably (which would have given us a PPF that was a straight line from the first to the last point).

3. For each of the following Cobb-Douglas utility functions, convert it to the simplest possible "logarithmic utility" version (recalling that the two are ordinally equivalent).

   (a) $u(x, y) = x^{\frac{3}{4}} y^{\frac{1}{4}}$

   (b) $u(x, y) = (2x)^{\frac{3}{4}} y^{\frac{1}{4}}$

   (c) $u(x, y) = x^{\frac{3}{4}} y^{\frac{3}{4}}$

4. Alvin and Beatrice both love chips (x) and pickles (y). Their utility functions are:

$$u^A(x^A, y^A) = \ln(x^A) + 4\ln(y^A) \qquad \text{and} \qquad u^B(x^B, y^B) = 2\ln(x^B) + \ln(y^B)$$

   Alvin and Beatrice have the same endowments, $e^A = e^B = (50, 50)$.

   (a) Draw the Edgeworth box and the endowment point.

   (b) Find the formula for the contract curve and plot it in the Edgeworth box (just sketch roughly what it looks like, perhaps using a graphing calculator or www.wolframalpha.com if necessary to get an idea of the shape). You may also need the following derivatives:

$$\frac{d}{dx}\left(a\ln(x) + b\ln(y)\right) = \frac{a}{x} \qquad \text{and} \qquad \frac{d}{dy}\left(a\ln(x) + b\ln(y)\right) = \frac{b}{y}$$

   (c) Solve for the equilibrium allocation and prices and plot the allocation on the Edgeworth box. You may want to use the following trick for Cobb-Douglas demand functions: if utility is $u^i(x^i, y^i) = a\ln(x^i) + b\ln(y^i)$ and agent i has income w^i, her demand functions are:

$$x^i = \frac{a}{a+b}\frac{w^i}{p_x} \qquad \text{and} \qquad y^i = \frac{b}{a+b}\frac{w^i}{p_y}.$$

   Note: p_y and the equilibrium allocation involve some weird fractions.