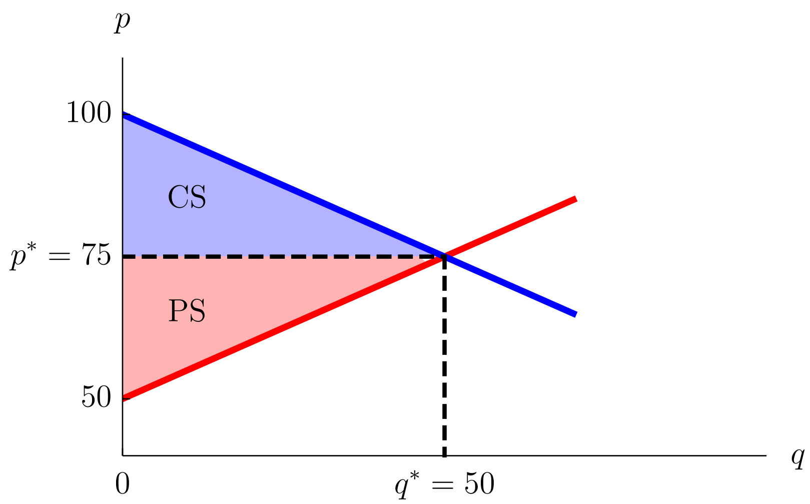
ECON 101, Problem Set 3
Due Tuesday, June 28

1. Suppose market supply is $q_S = 2p - 100$ and market demand is $q_D = 200 - 2p$.

   (a) Find market-clearing price and quantity. Plot the curves and label producer and consumer surplus.

   Solution:



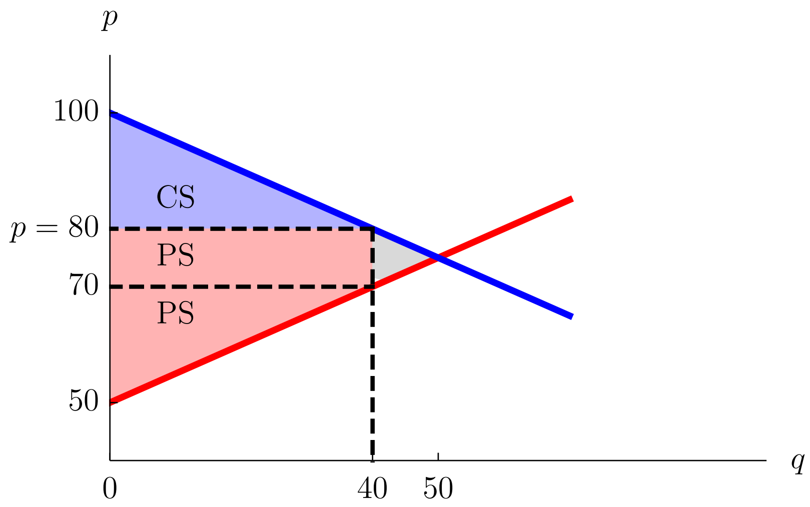
   (b) Calculate producer and consumer surplus.

   Solution:

   Consumer surplus is the area of the triangle with height 25 and base of 50: $CS = \frac{25 \cdot 50}{2} = 625$. Producer surplus has the same area.

   (c) Now suppose the government mandates a price floor of 80. Draw a new plot to reflect these changes, showing the new surpluses and deadweight loss.

   Solution:



   (d) Calculate the new surpluses and deadweight loss.

   Solution:

Consumer surplus is the area of the triangle with height 20 and base of 40: $CS = \frac{20 \cdot 40}{2} = 400$. Producer surplus is the area of a rectangle with height 10 and base 40 added to the area of a triangle with height 20 and a base of 40: $PS = 10 \cdot 40 + \frac{20 \cdot 40}{2} = 800$. Since the total surplus with the market prices was 1250, the DWL must equal $1250 - 800 - 400 = 50$. Could also explicitly calculate the area of the triangle: $DWL = \frac{10 \cdot 10}{2} = 50$.

2. Nelson runs a company that makes chips (x) and pickles (y). He has four employees:

   - Anne can make 5 chips or 3 pickles per hour.
   - Bob can make 4 chips or 2 pickles per hour.
   - Cathy can make 3 chips or 5 pickles per hour.
   - Derek can make 2 chips or 4 pickles per hour.

   Each of the employees works a ten-hour day.

   (a) Who has the absolute advantage in each of the goods?

   Solution:

   Anne has the absolute advantage in chips. Cathy has the absolute advantage in pickles.

   (b) Rank the workers based upon their comparative advantage in pickles.

   Solution:

   Derek has the comparative advantage in pickles as he only foregos 2/4 bags of chips per pickle. Cathy is second as she only foregos 3/5 bags of chip per pickle. Then Anne, who foregos 5/3 bags of chips per pickle. And finally Bob who foregos 4/2 bags of chips per pickle. Note that:

   $$\frac{2}{4} < \frac{3}{5} < \frac{5}{3} < \frac{4}{2}$$

   (c) If all four workers work on pickles, how many pickles will they make in one day.

   Solution:

   They would produce $(3 + 2 + 5 + 4) \cdot 10 = 140$ pickles.

   (d) If three workers work on pickles, which three should it be. What is the resulting output of chips and pickles?

   Solution:

   It should be everybody but Bob, as he has the comparative advantage in chips. The resulting output is $4 \cdot 10 = 40$ chips and $(3 + 5 + 4) \cdot 10 = 120$ pickles.

   (e) If two workers work on pickles, which two should it be. What is the resulting output of chips and pickles?

   Solution:

   Now it should be Derek and Cathy making pickles as they forego the least chips per pickle. The resulting output is $(5+4) \cdot 10 = 90$ chips and $(5+4) \cdot 10 = 90$ pickles.

   (f) If one worker works on pickles, which one should it be. What is the resulting output of chips and pickles?

Solution:

If it's just one worker, it should be Derek making pickles. The resulting output is $(5+4+3)\cdot 10 = 120$ chips and $4 \cdot 10 = 40$ pickles.

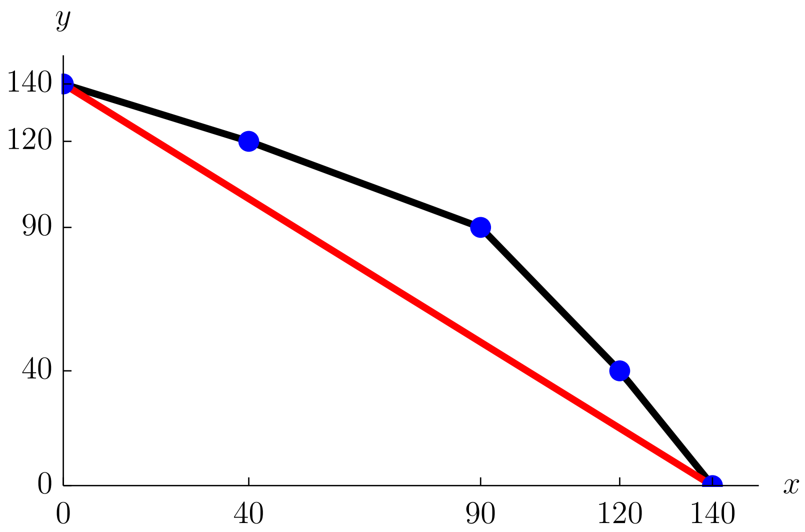
(g) If no worker works on pickles, how many chips will they make in one day?

Solution:

The resulting output is $(5+4+3+2)\cdot 10 = 140$ chips.

(h) Plot the five points you just found in (x, y) space. Connect the dots with straight-lines. If you have done it correctly, this is Nelson's PPF. Observe that using the workers according to their comparative advantage allows us to produce more output than if we had used them interchangeably (which would have given us a PPF that was a straight line from the first to the last point).

Solution:



Note that a correct PPF will always be concave. That is, the slope of each segment (going left to right) is steeper than the slope of the segment before. If yours does not do that, then you haven't used the workers correctly. Also remember that productive efficiency requires us to produce on the PPF, i.e. the black line above.

3. For each of the following Cobb-Douglas utility functions, convert it to the simplest possible "logarithmic utility" version (recalling that the two are ordinally equivalent).

(a) $u(x, y) = x^{\frac{3}{4}} y^{\frac{1}{4}}$

Solution:

We can take as many monotone increasing transformations as we want and we'll still have an ordinally equivalent utility function ($\hat{u}$).

$$\begin{aligned}\hat{u}(x, y) &= \ln\left(x^{\frac{3}{4}} y^{\frac{1}{4}}\right) \\ &= \ln\left(x^{\frac{3}{4}}\right) + \ln\left(y^{\frac{1}{4}}\right) \\ &= \frac{3}{4}\ln(x) + \frac{1}{4}\ln(y)\end{aligned}$$

Taking another increasing monotone transformation (quadrupling):

$$= 3\ln(x) + \ln(y)$$

(b) $u(x, y) = (2x)^{\frac{3}{4}} y^{\frac{1}{4}}$

Solution:

Exact same thing as above. Why? Because we can rewrite

$$(2x)^{\frac{3}{4}} y^1 4 = 2^{\frac{3}{4}} x^{\frac{3}{4}} y^{\frac{1}{4}}$$

And then the first monotone transformation we do is to multiply by $\frac{1}{2^{\frac{3}{4}}}$ and we're back to where we started the last one.

(c) $u(x, y) = x^{\frac{3}{4}} y^{\frac{3}{4}}$

Solution:

We can take as many monotone increasing transformations as we want and we'll still have an ordinally equivalent utility function ($\hat{u}$).

$$\begin{aligned}\hat{u}(x, y) &= \ln\left(x^{\frac{3}{4}} y^{\frac{3}{4}}\right) \\ &= \ln\left(x^{\frac{3}{4}}\right) + \ln\left(y^{\frac{3}{4}}\right) \\ &= \frac{3}{4}\ln(x) + \frac{3}{4}\ln(y)\end{aligned}$$

Taking another increasing monotone transformation (multiplying by 4/3):

$$= \ln(x) + \ln(y)$$

4. Alvin and Beatrice both love chips (x) and pickles (y). Their utility functions are:

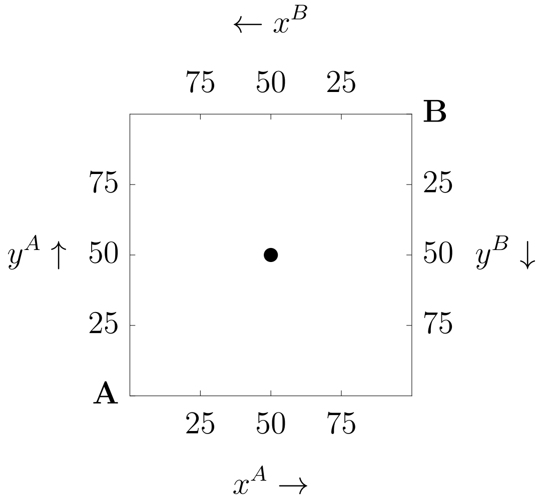
$$u^A(x^A, y^A) = \ln(x^A) + 4\ln(y^A) \qquad \text{and} \qquad u^B(x^B, y^B) = 2\ln(x^B) + \ln(y^B)$$

Alvin and Beatrice have the same endowments, $e^A = e^B = (50, 50)$.

(a) Draw the Edgeworth box and the endowment point.

Solution:

Now the Edgeworth box is a square because there are equal quantities of the two goods. Since they have the same endowments, the endowment point is right in the middle of the Edgeworth box.



(b) Find the formula for the contract curve and plot it in the Edgeworth box (just sketch roughly what it looks like, perhaps using a graphing calculator or www.wolframalpha.com if necessary to get an idea of the shape). You may also need the following derivatives:

$$\frac{d}{dx}\left(a\ln(x)+b\ln(y)\right)=\frac{a}{x} \qquad \text{and} \qquad \frac{d}{dy}\left(a\ln(x)+b\ln(y)\right)=\frac{b}{y}$$

Solution:

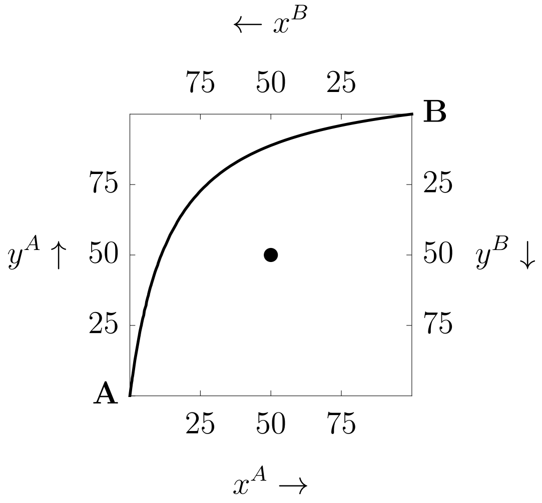
The contract curve, which shows all Pareto efficient points, can be solved by equating the MRS's of the two consumers.

$$\begin{aligned} MRS^A_{XY}(x^A,y^A) &= MRS^B_{XY}(x^B,y^B) \\ \frac{1/x^A}{4/y^A} &= \frac{2/x^B}{1/y^B} \end{aligned}$$

But note that, optimally, $x^B=100-x^A$ and $y^B=100-y^A$:

$$\begin{aligned} \frac{1/x^A}{4/y^A} &= \frac{2/(100-x^A)}{1/(100-y^A)} \\ \frac{1}{x^A(100-y^A)} &= \frac{8}{y^A(100-x^A)} \\ y^A(100-x^A) &= 8x^A(100-y^A) \\ 100y^A-y^Ax^A &= 800x^A-8x^Ay^A \\ 7y^Ax^A+100y^A &= 800x^A \\ y^A(7x^A+100) &= 800x^A \\ y^A &= \frac{800x^A}{7x^A+100} \end{aligned}$$

If we plot that on the Edgeworth box, we get the black curve:



When the two agents have the same utility Cobb-Douglas utility function, the contract curve is just the diagonal line. Here, as you can see from the utility functions, Alvin puts more weight on consumption of y and Beatrice puts more weight on consumption of x, so the Pareto efficient points are up and to the left of the diagonal – these points have more y for Alvin and more x for Beatrice.

(c) Solve for the equilibrium allocation and prices and plot the allocation on the Edgeworth box. You may want to use the following trick for Cobb-Douglas demand functions: if utility is $u^i(x^i, y^i) = a\ln(x^i) + b\ln(y^i)$ and agent i has income w^i, her demand functions are:

$$x^i = \frac{a}{a+b}\frac{w^i}{p_x} \quad \text{and} \quad y^i = \frac{b}{a+b}\frac{w^i}{p_y}.$$

Note: p_y and the equilibrium allocation involve some weird fractions.

Solution:

We'll follow the steps laid out in the lecture:

Step 1: Normalize $p_x = 1$.

Step 2: Work out incomes as a function of p_y. Since the endowments are the same, the incomes are also the same:

$$w^A = w^B = 50p_x + 50p_y = 50 + 50p_y$$

Step 3: Work out demand for good y for each agent:

$$y^A = \frac{b}{a+b}\frac{w^A}{p_y} = \frac{4}{5}\left(\frac{50+50p_y}{p_y}\right)$$
$$y^B = \frac{b}{a+b}\frac{w^B}{p_y} = \frac{1}{3}\left(\frac{50+50p_y}{p_y}\right)$$

Step 3: Use market-clearing of good y to determine market p_y:

$$\begin{aligned} y^A + y^B &= 100 \\ \frac{4}{5}\left(\frac{50+50p_y}{p_y}\right) + \frac{1}{3}\left(\frac{50+50p_y}{p_y}\right) &= 100 \\ \frac{170}{3}p_y + \frac{170}{3} &= 100p_y \\ \frac{170}{3} &= \frac{130}{3}p_y \\ p_y &= \frac{17}{13} \end{aligned}$$

Step 4: Plug that price back into A's demand for y to get y^A:

$$\begin{aligned} y^A = \frac{b}{a+b}\frac{w^A}{p_y} &= \frac{4}{5}\left(\frac{50+50p_y}{p_y}\right) \\ &= \frac{4}{5}\left(\frac{50+50(17/13)}{(17/13)}\right) = \frac{1200}{17} \end{aligned}$$

Step 5: Plug this into the budget constraint (or p_y into the demand function x^A) to get x^A:

$$\begin{aligned} x^A + \left(\frac{17}{13}\right)\left(\frac{1200}{17}\right) &= 50 + \left(\frac{17}{13}\right)50 \\ x^A &= \frac{300}{13} \end{aligned}$$

Finally, we can plot $(x^A, y^A) = \left(\frac{300}{13}, \frac{1200}{17}\right)$ with a green point on the Edgeworth box: