ECON 101, Problem Set 4
Due Tuesday, July 5

1. Suppose there is an industry with an oligopoly of three firms (Firm A, Firm B, and Firm C). The inverse market demand is $p(q) = 300 - q$. Each firm has a marginal cost of zero. Derivatives that may be useful: if a is independent of variable x:

$$\frac{d}{dx}(a \cdot x) = a \quad \text{and} \quad \frac{d}{dx}(x^a) = a \cdot x^{a-1}$$

   (a) What type of industry might have firms with zero marginal cost? Explain your answer in a sentence or two.

   (b) What is the societal optimal quantity of production?

   (c) How much would each firm produce if the three were acting as a cartel? What would each firm's profits be?

   (d) Assuming Firm B and Firm C are behaving (i.e. producing q_B and q_C that you found in (c)), what is Firm A's optimal production q_A? What are firm A's profits if it betrays the cartel to produce this quantity.

   (e) How much would each firm produce in the Cournot-Nash equilibrium? Recall that to do this, you need to find a best response for each firm and then solve the three equations simultaneously (or you can cheat and use the symmetry to assume $q_A = q_B = q_C$ to avoid having to solve the three-equation system). What are each firm's profits in the Cournot-Nash equilibrium?

2. Suppose there is an industry where for $p > 50$, market demand is $q(p) = 0$ and for $p \leq 50$, $q(p) = 100 - p$. There are two firms, Firm A and Firm B. Firm A has $MC_A(q_A) = 10$ and Firm B has $MC_B(q_B) = 20$.

   (a) Plot the marginal social benefit and the marginal social cost as a function of q, assuming efficient production.

   (b) What is the socially optimal level of production? How much of that quantity should each firm produce?

   (c) Suppose Firm A and Firm B compete in Bertrand competition. Assume that firms can charge only integer values (i.e. whole numbers) and firms would rather produce for the whole market at $p_i = MC_i$ (yielding zero profits) than not produce (also yielding zero profits). Write each firm's best response function and plot them together in (p_A, p_B)-space.

   (d) What are the equilibrium prices charged by the two firms and each firm's profits?

3. Consider the following 3-player game where P1 chooses the row, P2 chooses the column, and P3 chooses the bimatrix (A or B). In each cell, P1's payoff is listed first, P2's is listed second, and P3's is listed third. Find the pure-strategy NE. Hint: It's the same as doing it for 2-player except now you have to find (for each player) the best response to each possible profile of opponent actions, underlining utilities accordingly. Note that if there are ever two best responses (two actions that do equally well against a particular profile of opponent actions) underline both of the corresponding utilities.

	L	R
U	$2, 0, 1$	$1, 1, 1$
D	$1, 1, 1$	$0, 2, 2$

A

	L	R
U	$0, 2, 2$	$1, 1, 2$
D	$1, 1, 1$	$2, 0, 1$

B

4. For each of the following infinitely repeated games, what is the minimum δ such that the grim-trigger strategy profile (which results in (C,C) forever) is a Nash equilibrium?

   (a) The infinitely repeated game with the following stage game:

	C	D
C	2,2	0,4
D	4,0	1,1

   (b) The infinitely repeated game with the following stage game:

	C	D
C	1,1	-1,4
D	4,-1	0,0