ECON 101, Problem Set 4
Due Tuesday, July 5

1. Suppose there is an industry with an oligopoly of three firms (Firm A, Firm B, and Firm C). The inverse market demand is $p(q) = 300 - q$. Each firm has a marginal cost of zero. Derivatives that may be useful: if a is independent of variable x:

$$\frac{d}{dx}(a \cdot x) = a \quad \text{and} \quad \frac{d}{dx}(x^a) = a \cdot x^{a-1}$$

(a) What type of industry might have firms with zero marginal cost? Explain your answer in a sentence or two.

Solution:

Digital industries often have a marginal cost approaching zero. How much does it cost Beyonce to have one more user download her song? How much does it cost the *New York Times* to have one more reader read an article online? In both cases, the answer is zero.

(b) What is the societal optimal quantity of production?

Solution:

Societal optimal production is up until the unit where the marginal social benefit equals the marginal social cost. Here the marginal social cost is just the marginal cost which is zero and the marginal social benefit is just the inverse market demand curve. Therefore, the societal optimal quantity q^* solves $300 - q^* = 0$, i.e. $q^* = 300$.

(c) How much would each firm produce if the three were acting as a cartel? What would each firm's profits be?

Solution:

The revenue for the cartel is $q \cdot p(q) = q(300 - q) = 300q - q^2$. Using the derivatives above to find the marginal revenue gives us $MR(q) = 300 - 2q$. The cartel solves $MR(q) = MC(q)$: $300 - 2q = 0$ so $q^{\text{Cartel}} = 150$. Each firm would produce $q_A = q_B = q_C = 50$ units. Profits for firm i are:

$$p(q) \cdot q_i = 150 \cdot 50 = 7500.$$

Since q_i is the same for $i = A, B, C$, each firm has profits 7500.

(d) Assuming Firm B and Firm C are behaving (i.e. producing q_B and q_C that you found in (c)), what is Firm A's optimal production q_A? What are firm A's profits if it betrays the cartel to produce this quantity.

Solution:

Firm A's revenue is:

$$q_A \cdot p(q) = q_A \cdot p(q_A + q_B + q_C) = q_A(300 - q_A - 100) = 200q_A - q_A^2$$

Then marginal revenue is $200 - 2q_A$. Setting this equal to marginal cost would yield $q_A = 100$. If firm A does produce 100 units then the $p(q) = p(200) = 300 - 200 = 100$ and firm A's profits will be $100 \cdot 100 = 10000$.

(e) How much would each firm produce in the Cournot-Nash equilibrium? Recall that to do this, you need to find a best response for each firm and then solve the three equations simultaneously (or you can cheat and use the symmetry to assume $q_A = q_B = q_C$ to avoid having to solve the three-equation system). What are each firm's profits in the Cournot-Nash equilibrium?

Solution:

Firm A's revenue (as a function of q_B and q_C) is:

$q_A \cdot p(q_A + q_B + q_C) = q_A(300 - q_A - q_B - q_C) = 300q_A - q_A^2 - q_Aq_B - q_Aq_C$.

Then firm A's marginal revenue (differentiating with respect to q_A) is

$$300 - 2q_A - q_B - q_C.$$

Setting this equal to marginal cost gives:

$$\begin{aligned} 300 - 2q_A - q_B - q_C &= 0 \\ q_A &= \frac{300 - q_B - q_C}{2} \end{aligned}$$

Doing this for Firm B and Firm C also would give:

$$\begin{aligned} q_B &= \frac{300 - q_A - q_C}{2} \\ q_C &= \frac{300 - q_A - q_B}{2} \end{aligned}$$

You could solve all three of these simultaneously or assume symmetry ($q_A = q_B = q_C$) and use that on the first equation:

$$\begin{aligned} q_A &= \frac{300 - 2q_A}{2} \\ q_A &= 150 - q_A \\ q_A &= 75 \end{aligned}$$

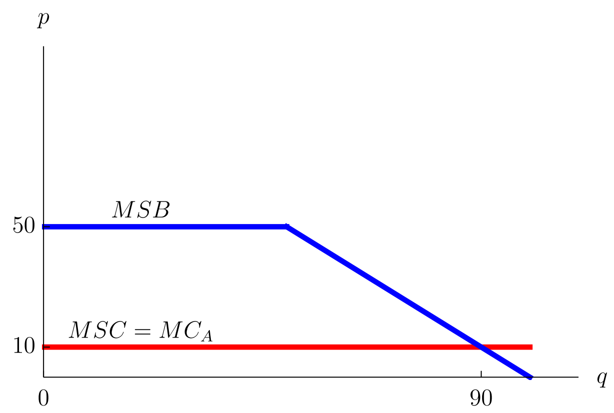
From that and symmetry we get $q_A = q_B = q_C = 75$ so total quantity is $q = 225$. So $p(q) = 75$ and profits for each firm are $75 \cdot 75 = 5625$.

2. Suppose there is an industry where for $p > 50$, market demand is $q(p) = 0$ and for $p \leq 50$, $q(p) = 100 - p$. There are two firms, Firm A and Firm B. Firm A has $MC_A(q_A) = 10$ and Firm B has $MC_B(q_B) = 20$.

   (a) Plot the marginal social benefit and the marginal social cost as a function of q, assuming efficient production.

   Solution:

   The marginal social benefit is a little weird, but not too difficult. To work out the marginal social cost, assuming efficient production, you simply need to note that it is inefficient for Firm B to produce any quantity as Firm A can produce more efficiently. Therefore, the marginal social cost assuming efficient production is just MC_A.



(b) What is the socially optimal level of production? How much of that quantity should each firm produce?

Solution:

The socially optimal level of production is $q = 90$. Firm A should produce all of it as it is more efficient.

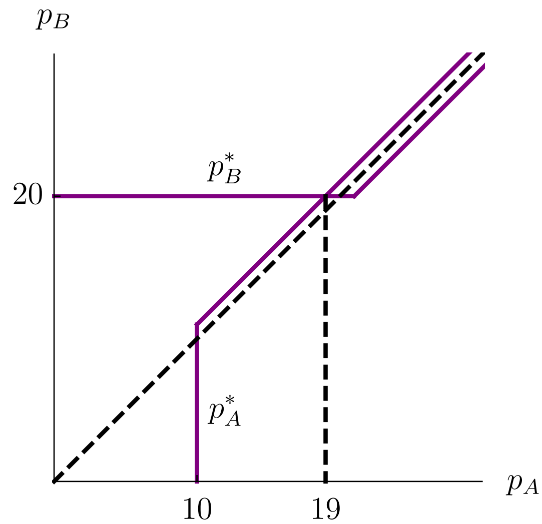
(c) Suppose Firm A and Firm B compete in Bertrand competition. Assume that firms can charge only integer values (i.e. whole numbers) and firms would rather produce for the whole market at $p_i = MC_i$ (yielding zero profits) than not produce (also yielding zero profits). Write each firm's best response function and plot them together in (p_A, p_B)-space.

Solution:

If the other firm is charging at least one greater than our marginal cost, our optimal price is to undercut them by one dollar. If the other firm is charging our marginal cost or below, our best response is to charge our marginal cost (or anything above).

$$p_A^* = \begin{cases} p_B - 1 & \text{if } p_B \geq 11 \\ 10 & \text{if } p_B < 11 \end{cases}$$

$$p_B^* = \begin{cases} p_A - 1 & \text{if } p_A \geq 21 \\ 20 & \text{if } p_A < 21 \end{cases}$$



(d) What are the equilibrium prices charged by the two firms and each firm's profits?

Solution:

Firm A charges $p_A = 19$ and Firm B charges $p_B = 20$. You might think that it doesn't matter what Firm B is charging, as it's not going to sell to anybody. But, actually, if Firm B is charging $p_B > 20$, then this isn't an equilibrium as Firm A would want to raise its price. As for the profits, obviously Firm B's profits are zero because it doesn't sell any units. As for Firm A, demand at $p = 19$ is $q(19) = 100 - 19 = 81$. So Firm A's profits (factoring in costs also) are $81(19 - 10) = 729$.

3. Consider the following 3-player game where P1 chooses the row, P2 chooses the column, and P3 chooses the bimatrix (A or B). In each cell, P1's payoff is listed first, P2's is listed second, and P3's is listed third. Find the pure-strategy NE. Hint: It's the same as doing it for 2-player except now you have to find (for each player) the best response to each possible profile of opponent actions, underlining utilities accordingly. Note that if there are ever two best responses (two actions that do equally well against a particular profile of opponent actions) underline both of the corresponding utilities.

	L	R
U	$2, 0, 1$	$1, 1, 1$
D	$1, 1, 1$	$0, 2, 2$

A

	L	R
U	$0, 2, 2$	$1, 1, 2$
D	$1, 1, 1$	$2, 0, 1$

B

Solution:

First, for every profile of P2 and P3 actions (LA,RA,LB,RB), underline the utility P1 gets from his best response:

	L	R
U	$\underline{2}, 0, 1$	$\underline{1}, 1, 1$
D	$1, 1, 1$	$0, 2, 2$

A

	L	R
U	$0, 2, 2$	$1, 1, 2$
D	$\underline{1}, 1, 1$	$\underline{2}, 0, 1$

B

Then, for every profile of P1 and P3 actions (UA,DA,UB,DB), underline the utility P2 gets from his best response:

	L	R
U	$\underline{2}, 0, 1$	$\underline{1}, \underline{1}, 1$
D	$1, 1, 1$	$0, \underline{2}, 2$

A

	L	R
U	$0, \underline{2}, 2$	$1, 1, 2$
D	$\underline{1}, \underline{1}, 1$	$\underline{2}, 0, 1$

B

Finally, for every profile of P1 and P2 actions (UL,UR,DL,DR), underline the utility P3 gets from his best response:

	L	R
U	$\underline{2}, 0, 1$	$\underline{1}, \underline{1}, 1$
D	$1, 1, \underline{1}$	$0, \underline{2}, \underline{2}$

A

	L	R
U	$0, \underline{2}, \underline{2}$	$1, 1, \underline{2}$
D	$\underline{1}, \underline{1}, \underline{1}$	$\underline{2}, 0, 1$

B

The unique NE is (D,L,B).

4. For each of the following infinitely repeated games, what is the minimum δ such that the grim-trigger strategy profile (which results in (C,C) forever) is a Nash equilibrium?

 (a) The infinitely repeated game with the following stage game:

	C	D
C	2,2	0,4
D	4,0	1,1

Solution:

If we do not deviate, we get (C,C) continues forever, and the value of that is $\frac{2}{1-\delta}$. If we do deviate to D, we get 4 today but then play (D,D) forever, and the value of that is $4 + \delta\frac{1}{1-\delta}$. Therefore, we are willing to not deviate so long as:

$$\begin{aligned} \frac{2}{1-\delta} &\geq 4 + \delta\frac{1}{1-\delta} \\ 2 &\geq 4(1-\delta) + \delta \\ 2 &\geq 4 - 3\delta \\ \delta &\geq \frac{2}{3} \end{aligned}$$

Therefore the minimum δ such that grim-trigger is a NE of the infinitely repeated game is 2/3.

 (b) The infinitely repeated game with the following stage game:

	C	D
C	1,1	-1,4
D	4,-1	0,0

Solution:

If we do not deviate, we get (C,C) continues forever, and the value of that is $\frac{1}{1-\delta}$. If we do deviate to D, we get 4 today but then play (D,D) forever, and the value of that is $4 + 0$. Therefore, we are willing to not deviate so long as:

$$\begin{aligned} \frac{1}{1-\delta} &\geq 4 \\ 1 &\geq 4 - 4\delta \\ \delta &\geq \frac{3}{4} \end{aligned}$$

Therefore the minimum δ such that grim-trigger is a NE of the infinitely repeated game is 3/4.