ECON 101, Problem Set 5
Due Tuesday, July 12

1. Bob is founding a broadcast television company. He will broadcast free television channels in Astana. Let x denote the number of channels he decides to provide. Suppose his marginal cost of creating each channel is ten million dollars and that his marginal personal benefit is $MPB(x) = 1000/x$. You cannot create a fraction of a channel, i.e. x only takes integer values.

    (a) How many channels will Bob create?

    Solution:

    None. The marginal cost of the first channel is ten million and the marginal private benefit is only one thousand.

    (b) If there are a million people (including Bob) that have identical MPB's to Bob, what is the socially optimal number of channels?

    Solution:

    The marginal social benefit is now Bob's MPB multiplied by a million. Equating that to marginal social cost (ten million) gives:

    $$1\,000\,000 \cdot \frac{1000}{x} = 10\,000\,000 \qquad \Longrightarrow \qquad x^* = 100$$

    (c) If the government wants to induce the socially optimal outcome, how much of a subsidy should it give Bob for each channel he creates?

    Solution:

    To get Bob to internalize the externality, the government should bring his MPB up to the MSB through a subsidy. Therefore, for the xth channel, they should give him a subsidy of $999\,999 \cdot \frac{1000}{x}$. That is, the subsidy is equal to the marginal external benefit. Note that the subsidy for the first channel is the highest and it gets lower for each additional channel.

2. Suppose we have the following taxes:

    - A progressive income tax, with the following tax brackets and deductions:
        - $[0, \$20000] \to 10\%$
        - $(\$20000, \$40000] \to 20\%$
        - $(\$40000, \infty) \to 30\%$
        - All childcare expenses deductible.
        - Each person may deduct $1000 per alligator in household.
    - A sales tax of 10% on all purchases (except childcare which is exempt). The sales tax is included in purchase prices.

    (a) Al has an income of $45000. He has two alligators and spends no money on childcare. He spends 50% of his after-tax income on consumption (NOT including the sales taxes). What is the total amount that he paid to the government in taxes?

Solution:

The total income tax he pays is

$$20000 \cdot 0.1 + 20000 \cdot 0.2 + 3000 \cdot 0.3 = 6900$$

Note that we gave him the alligator deduction – that's why it's $3000 \cdot 0.3$ instead of $5000 \cdot 0.3$. This means that his after-tax income is $45000 - 6900 = 38100$. He spent half of that on consumption, i.e. 19050. And then he paid 10% of that total in sales taxes. So he paid a total of 1905 in sales tax. His total tax paid is then:

$$6900 + 1905 = 8805$$

(b) Beth also has an income of $45000. She spent $5000 on child care and has no alligators. She paid $500 in sales tax. What are her savings (i.e. how much is left after taxes and consumption)?

Solution:

Since she spent 5000 on child care, her total income tax bill is just $20000 \cdot 0.1 + 20000 \cdot 0.2 = 6000$. Her after tax income is $45000 - 6000 = 39000$. She spent 5000 on childcare, leaving her with 34000. Finally, if she paid 500 in sales taxes, she must have spent 5000 on consumption (NOT including the taxes), so her savings are 28500.

(c) Colin earns $28000. His consumption not including childcare and the sales tax was $5000. He has one alligator. The total tax he paid to the government was $3000. How much did he spend on childcare? *Hint: it is less than $7000.*

Solution:

From his consumption, we know he paid 500 for sales taxes. So he must have paid 2500 in income tax. Let x denote the amount he spent on childcare, then we know that:

$$20000 \cdot 0.1 + (8000 - 1000 - x) \cdot 0.2 = 2500$$

Solving this reveals that he must have spent $x = 4500$ on childcare expenses.

(d) Deborah earns $45000. She has 20 alligators and spends 5000 on childcare. What is her marginal income tax rate? What about her average income tax rate?

Solution:

Her marginal income tax rate is 20%. After you apply all of her deductions, her taxable income is 20000. The next dollar she earns will be taxed at the 20% rate.

For the average income tax rate, you could divide the income tax she pays (2000) by her income (45000) to get about 4.4%, which of course is lower than even the lowest rate because of all of the deductions. Technically, however, you should divide not by her total income but by her taxable income as the deductions are a modifiction to the tax base, not the rate structure. So then it would be 2000/20000, or 10%. Either answer is fine.

3. Suppose the market inverse demand for peaches is $p(q) = 100 - q/2$, there is perfect competition on the supply side, and each firm's marginal cost is $MC(q) = q/3$.

(a) If the government taxes consumers with a \$20 tax per unit consumed, what is the new market outcome? Calculate CS, PS, TR and DWL. How much of the tax is paid by the consumers? How much by the producers? Show CS, PS, TR, DWL and the appropriate curves on a plot in (q, p)-space.

Solution:
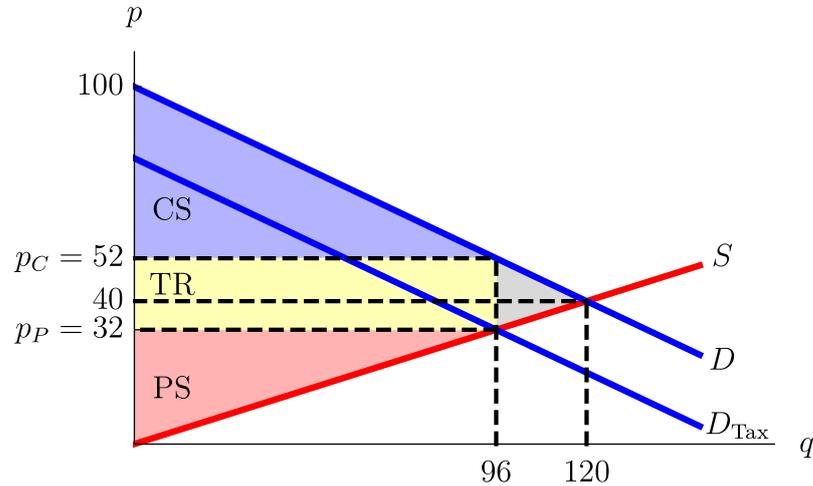
- New market outcome is $q = 96$ and $p = 32$, though consumers face an effective price of $p_P = 52$, including the tax.

- Consumer surplus is $\frac{48 \cdot 96}{2} = 2304$.

- Producer surplus is $\frac{32 \cdot 96}{2} = 1536$.

- Tax revenue is $20 \cdot 96 = 1920$.

- Deadweight loss is $\frac{20 \cdot 24}{2} = 240$

- Consumers pay $12 \cdot 96 = 1152$ of the tax.

- Producers pay $8 \cdot 96 = 768$ of the tax.

Note that producers pay less of the tax because their supply curve is more elastic (i.e. less steep) than the demand curve.

(b) If the government taxes producers with a \$20 tax per unit produced, what is the new market outcome? Calculate CS, PS, TR and DWL. How much of the tax is paid by the consumers? How much by the producers? Show CS, PS, TR, DWL and the appropriate curves on a plot in (q, p)-space.

Solution:
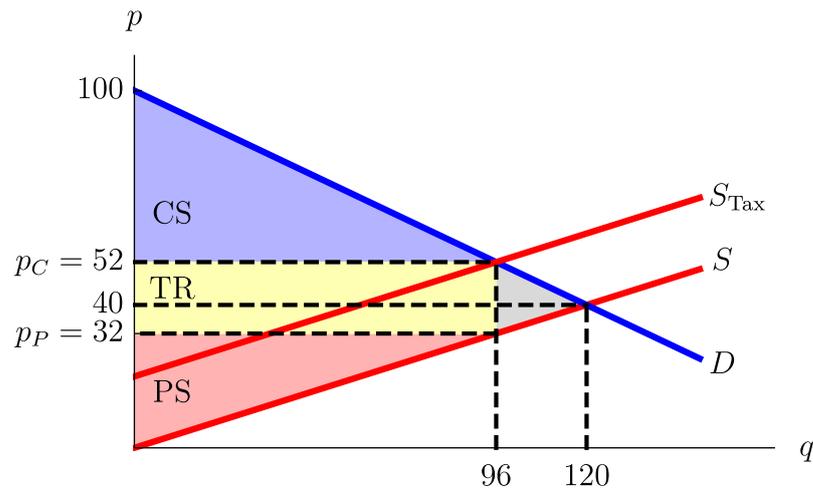
New market outcome is $q = 96$ and $p = 52$, though producers face an effective price of $p_P = 32$, including the tax. All other answers are identical to above.

(c) Suppose the government taxes all other food products at about the same rate as they tax peaches. Using the theory of the second best, give a qualitative argument that getting rid of just the taxes on peaches might be a bad idea for society.

Solution:

If there were just a distortionary tax on peaches, than we would be producing (and eating) too few peaches as a society. But given there is roughly the same distortion on all food products, and our total consumption of food is basically fixed, removing the tax on just peaches would mean we produced (and ate) to many peaches as a society (relative to other food stuffs).

4. For each of the following statements, say whether it is true or false and give a short explanation of your answer.

(a) The principle of neutrality suggests that we should tax goods for which supply and demand are relatively elastic so that the tax is not distortionary.

Solution:

False. While the principle of neutrality does suggest we should avoid distortionary taxes, taxes on goods with elastic supply and demand are very distortionary. It's less distortionary to tax goods with inelastic supply and demand.

(b) If bargaining transaction costs are low, there is no role for government in correcting markets for externalities.

Solution:

This might appear true, and it's almost just a statement of the Coase theorem. But it's missing the assumption that trade in an externality is possible. And for that to be true, clear property rights must be defined and enforced by the government. So the statement is false – the government would still have a role in defining and enforcing property rights.

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(c) In the presence of a negative externality, the total surplus is identical whether the government simply mandates the socially optimal outcome or the government implements a Pigouvian tax that results in the socially optimal outcome.

Solution:

This is true. It will be divided up differently as the latter case will have tax revenue where the former won't, but the total surpluses will equate.

(d) If all people consume the same percentage of their income, a flat-rate sales tax (say 12% on all purchases) is a proportional tax.

Solution:

This is true. When we showed it was regressive in class, this was under the assumption that lower income families consumed a greater portion of their income than high-income families. Under these conditions, the tax was regressive. But if everybody is consuming the same percentage of their income, it's proportional because of the flat rate.